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Mathematics Teaching

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About *Mathematics Teaching*

Mathematics Teaching is the journal of the Association of Teachers of Mathematics (ATM). The ATM was formed in 1952 to encourage the teaching and learning of mathematics by relating closely to the needs of the learner. The aims and guiding principles of the ATM can be found on the back page of the journal.

Mathematics Teaching does not seek to conform to an official view on the teaching of mathematics, whatever that may be. We wish to encourage contributors to express their personal views on the teaching and learning of mathematics. We believe that everyone has a contribution to make and experiences and insights to share. Whether practical, political, philosophical or speculative, we are looking for articles that reflect on the practice of teaching mathematics. We aim to publish articles that will be of interest to the breadth of our membership, as well as a balance between those derived from research and those from practical experience. We see writing as a powerful tool for professional development and critical thinking and see Mathematics Teaching as a space for this development to take place.

The Editorial Board hopes that:

- Mathematics Teaching is a space to share investigations into our teaching in some detail. Not generalising too much, but speaking for ourselves and of our experience. Never trying to speak for all.
- Mathematics Teaching honours the different ways that people know, do and engage with mathematics.
- readers will learn from engaging with the articles, in trying things out and in noticing the impact of their new practice.
- readers will engage in dialogue with the articles between issues.
- Mathematics Teaching gives teachers confidence to use ideas that are current and relevant to them in the ever-changing world of teaching and learning mathematics.
- articles do not shy away from describing the tensions, imperfections and challenges that are a part of teaching and learning mathematics.

The focus of articles may be:

- an account of something mathematical.
- an account of a visit to another classroom, another school, or another country.
- a description of using a favourite piece of equipment or resource.
- a reflection on experiences of different teaching and learning styles.
- views and news on current initiatives.
- responses to, or reflections on, articles from previous editions of Mathematics Teaching.

The Editorial Board members are:

Tom Bennison is a Head of Sixth Form in Derbyshire. During his applied mathematics PhD he realised his passion was teaching and has taught secondary mathematics ever since. He is particularly interested in how the tasks chosen in the classroom develop students to think mathematically and be creative.

Tony Cotton (Editor) first joined the ATM in 1981. He has taught in secondary schools, spent time as an advisory teacher for anti-racist and multicultural education then worked in publishing before moving into teacher education. He worked in teacher education in Nottingham and Leeds before leaving to work as a writer and educational consultant.

Cath Gripton is an Associate Professor of Mathematics Education at the University of Nottingham. She is a researcher and former teacher educator, having previously taught 3 – 7 year olds in Nottingham schools for fourteen years.

Marc North is an Assistant Professor at the University of Nottingham, working as a tutor on the Primary Post Graduate Certificate of Education programme. He started out his career as a secondary mathematics teacher but has since worked in both primary and further education settings. He works actively with local partnership organisations to support teachers' professional learning, including East Midlands Maths Hubs and the Derby Research School. He is particularly interested in the role of contexts and representations for supporting mathematical learning. In his spare time, he pretends to be a serious cyclist.

Deliah Pawluch taught in Leicester primary schools for ten years, where she was drawn to the teaching and learning of mathematics by her learners' needs, Rose Griffiths' twilight professional development sessions, old copies of MT... and conversations with Derek Ball at her first ATM conference. After studying mathematics education with the Open University (with the help of her Year 6 classes), she joined the primary mathematics team at Nottingham Trent University (NTU).

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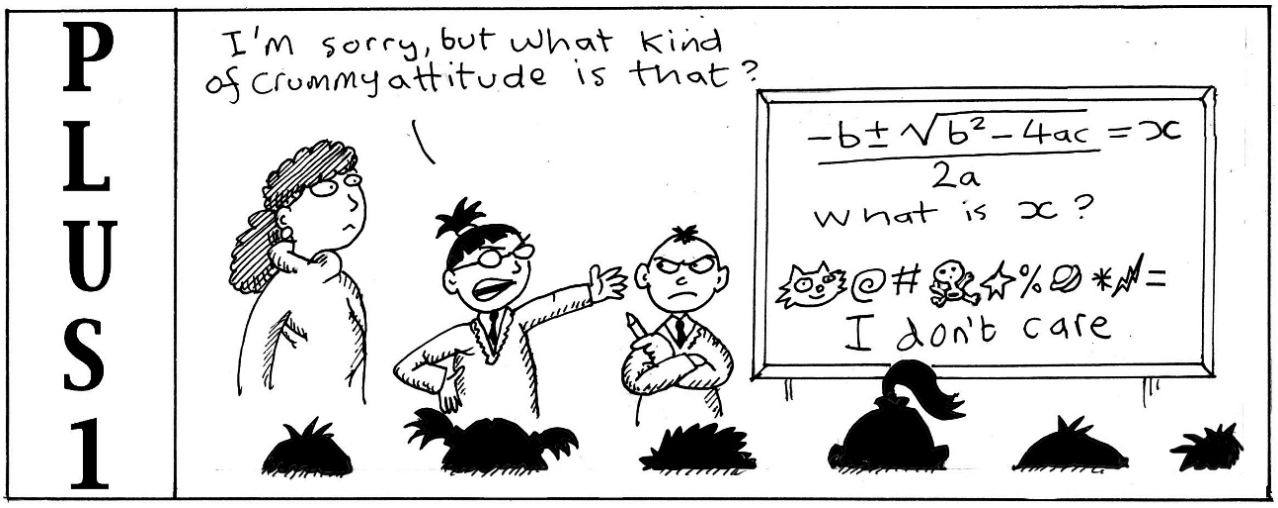

Plus 1 drawn by Harry Venning.

Cover 7:84 by Pete Moser

In the seventies this was the name of a political theatre company that toured the UK. 7% of the population own 84% of the wealth. Is this OK ? What is it now ? 1% own 70%. The imbalance of wealth in our society is a problem with many people living in food poverty whilst others have so much wealth that all they can do is store it in banks and in other ways that just increase its size.

Do we think this is right? What can we do about it?

The front cover vinyl graphic was created by Pete Moser - an activist artist in Morecambe.
www.petemoser.com

Forewords

Tony Cotton introduces this special issue, his final issue as editor.

Against homogeneity

> Imagine what would happen if we started treating children in accordance with the faith that they are equal in intelligence and able to learn without teaching! How wonderful, liberating and energising that would be! It could be our gift to children around the world who suffer from the myth of inequality, used as a basis for racism and for subduing people.

This is how Munir Fasheh concluded the opening article in MT250, my first issue as editor nearly nine years ago. I hope I have carried this image with me over the last nine years. Imagine if we acted on the ATM guiding principle, "The power to learn rests with the learner. Teaching has a subordinate role. The teacher has a duty to seek out ways to engage the power of the learner." Imagine the mathematics curriculum then. Imagine what we might expect our classrooms and learning spaces to look like. Imagine if the only similarity we would wish for in the learners in our care was their ability to learn. Imagine embracing difference to the extent that 'inequality' was impossible to define. Imagine.

I started writing these forewords in Bangkok where I teach on an international postgraduate qualification. The diversity on this course is beautiful. It is impossible to make assumptions about the participants' prior life experience and how this might impact on their values and beliefs about education. And this makes the course so much richer than courses I teach back in England, where, often, there is a shared common sense about how schools might operate. This is no fault in these participants but in a system which means many talented individuals are excluded from accessing teacher education by a system that limits opportunities from people with diverse backgrounds. If you are poor, if you have ways of being that are defined as disabilities by those who keep the gates closed, or if you are from communities which are sometimes defined as minorities (although, this of course depends on how you define the majority community) you will find it much harder to access further and higher education.

Anyway – back to Bangkok. During one of the breaks, I say to one of the participants, who, in her brief autobiography had described herself as French-Canadian, that I would ask the hotel staff to keep the buffet open. She immediately asked the member of staff to do this in fluent Thai. I said that I was pleased she had learned Thai during her time in Bangkok as many of the teachers who settle in Thailand do not. She said, "Tony, my mum is Thai/Cambodian/Chinese." It struck me that words such as ethnicity or culture become even more slippery here. And that this is a positive thing. Is this a vision of the future? – we can only hope.

I return to working on these final forewords in Vietnam after a harrowing visit to the war remnants museum in Ho Cho Minh City. I remember many of the images from the war in Vietnam from my childhood. I wonder if the experience of watching the war unfold on the television during my primary education informed both my politics and my beliefs. I also cannot help but make comparisons between this war and the current wars being waged in Ukraine and Gaza. The images from Gaza of cities and communities completely destroyed. I can but hope that my children and grandchildren can visit Gaza in 50 years time and experience such a vibrant and thriving city as I did in the current Ho Chi Minh City.

But, I hear you ask, "What has this got to do with mathematics education?" I am afraid that I think mathematics and the mathematics curriculum is culpable in stifling diversity. Many schools still implement the anachronistic and divisive system of grouping learners by prior attainment. Statements in the English national curriculum such as, "The expectation is that the majority of pupils will move through the programmes of study at broadly the same pace." stifles rather than encourages a diversity of approaches. And a harsh inspection regime that seems to aim to enforce a one-size-fits-all approach to learning and teaching mathematics. All of this leads to teachers feeling unable to explore mathematics in diverse and creative ways as they feel limited by school and government expectations. All of this aims for homogeneity.

Things can be different, and I hope this issue offers an alternative view of what schools might look like. The authors imagine what such classrooms can be like, they embrace difference, and most importantly these author/teachers create these spaces for learning through their own work. And, as Paolo Freire said, "in working we transform the world."

As Kate Kitigawa and Timothy Revell conclude their marvellous book *The secret lives of numbers: A global history of mathematics and its unsung trailblazers* in which they write:

> … as mathematics and the people pursuing it become more diverse, in every possible sense, so too will our means to tackle problems. … Progress will not be linear – it never has been. And it never should be. A diversity of ideas and approaches is what makes the human pursuit of knowledge so successful." (Page 262)

Reference

Kitigawa, K. and Revell, T. (2024) The secret lives of numbers: *A global history of mathematics and its unsung trailblazers.* London: Penguin Books.

Reclaiming what has been made invisible

Munir Fasheh, whose piece *Over 68 years with mathematics* opened Tony's first issue as editor (MT250), opens Tony's final issue as editor.

The following are mathematics courses that I have studied, and some of which I taught: differential equations, introduction to linear algebra, mathematical statistics, complex variables, discrete mathematics, topics in algebra, numerical analysis, theory of numbers, topology, and probability.

What is missing in these courses? Meaning, context, learning as a biological ability, nurturing soils, reflection, underlying logic, wisdom.

What remains? Illusions and mastering technical skills and meaningless symbols at best. I excelled in dealing with them as a student and then as a teacher, which gave me a sense of worth. It made me believe that I was intelligent. I received praise and was offered degrees, jobs, tenure. In fact, I excelled as a parrot repeating what is in textbooks and academic books and strived to be a copy of those whom I thought then to be superior intellectually and morally. The words in those courses invaded my mind and bombarded my thinking in a way that shattered the knowledge that is supposed to be within them. Courses mentioned above were 'Trojan Horses' that defeated me from within. My saviour was my mother who did not even know the alphabet. (See MT250 and 251 for an elaboration of this). The secret in her was in knowing reality with no filters (such as the alphabet). How can we talk about these courses as knowledge in the absence of aspects that I didn't realise until the 1967 Israeli-Arab war, which represented the first shock I encountered in relation to education, and mathematics in particular. I was then teaching mathematics at Birzeit College in the West Bank of Palestine (occupied by Israel). The war made me realize I was deceived because I was living in a manufactured world dominated by institutional terminology and academic categories. The manufactured world that I encountered in those courses made the real world invisible – a world that consists of living nurturing soils: earth, social, cultural, spiritual, and economic. How can knowledge happen outside those soils? They can if the knowledge under consideration is related to technical aspects, but not to those related to life.

The dresses my mother made were art, designed by her. Every dress she designed fitted the body of the woman who had brought the piece of cloth a few days earlier. As an academic, after realising that what my mother was doing was a form of mathematics that I could neither understand nor do, I did not have the courage, until 1984, to write about it, and until 1990 to publish this writing. The poverty and flatness in my knowledge were basically distractions. Without wisdom we accept absurd things, such as believing that the worth of a person can be expressed by a number or category, and that knowledge can be measured by credit hours (which may be sensible in technical fields, but not in life-related aspects). My knowledge in mathematics was an accumulation of credit hours required for graduation. Important aspects, like meaning and context, were not required; they are not crucial in academia. My mother's knowledge was in harmony with dignity; mine was degrading, by struggling to be a copy or parrot. I then started telling students not to accept anything that has no meaning and is not connected to context and action. I want to remind readers that I am not talking here about technical matters but about life-related areas. An organic teacher is one whose expressions come from within in relation to what is around. The words of a professional teacher come merely out of their tongue. Learning to humans is like wings to birds; if a bird loses its wings, it loses being a bird; *and*, if a person loses learning as an innate ability, they lose what it is to be fully human. Conversations and interactions with other children without interference from professionals are nurturing; children learn within live soils, such as conversations. It is a blessing. Conversation is a human miracle par excellence. We need to work very hard to keep it alive in as many settings as possible. Dogmatic curricula and devices such as mobiles rob children from being fully human.

At a completely different level, the issue I encountered in my teaching of mathematics is related to the two-valued logic that we teach in mathematics curricula as if it were the best. We are told that every statement is either true or false, with no third alternative; every student either succeeds or fails, no third alternative; every country is either developed or undeveloped, no third alternative. The two-valued logic is needed in inventing devices, machines, and controlling

people (through various means), but not in dealing with life. A feature that is related to logic, but made invisible, is related to the underlying logic that controls a phenomenon or is shared by different phenomena. Here are three examples that share the same underlying logic: formal education in India articulated by Macaulay in 1835, the establishment of the American University in Beirut by Daniel Bliss in 1866 and the concept of development as articulated by President Truman in the United States in 1949.

In 1835, Britain sent Thomas Macauley to India to develop a strategy of how to rule millions of people with a small number of individuals. This is what he wrote:

I have conversed … with men distinguished by their proficiency in Eastern tongues … I have never found one … who could deny that a single shelf of a good European library was worth the whole native literature of India and Arabia. The intrinsic superiority of Western literature … is fully admitted by [them]… We must… do our best to form a class who may be interpreters between us and the millions whom we govern; a class of persons, Indian in blood and colour, but English in taste, in opinions, in morals, and in intellect … we have to educate [them].

The underlying logic of his strategy is built on three premises: degrading, monopolizing, and helping. Degrading Indians, monopolising the solution (the British path), and ready to help Indians become copies of the British. It worked very well. Rishi Sunak (British Prime Minister 2022-2024) is a striking example.

Thirty years after Macaulay, in 1866, Daniel Bliss established the Protestant Syrian College (later, the American University in Beirut) which followed the same logic used by Macaulay. In his address 1871, Bliss declared:

This College is for all conditions and classes of men without regard to color, nationality, race, or religion… but it will be impossible for anyone to continue with us long without knowing what we believe to be the truth.

His logic degraded wisdom (which is the heart of the Arab Islamic civilisation for centuries) and replaced it by truth which he considered higher than wisdom, and dealt with truth as a monopolising path to be followed, and expressed his enthusiasm to help the Lebanese be like advanced Americans. (It is worth mentioning here that the motto of Harvard is VERITAS/ truth). If Bliss focused on technical matters only, that would have been fine. But claiming that his truth is universal, and he has the right to impose it, and the power to kick out from the university those who refuse to accept his truth, is a mischievous act that wipes out what characterised Arab civilization most. His allegiance is to America, which wiped out inhabitants of continents in the name of Jesus (whom Bliss was supposed to be devoted to). He could not see the crimes because that fact was made invisible. ‘Houses of wisdom’ spread from Isfahan in the east to Andalucia and Marrakesh in the west. The university Bliss established did good things connected to technical aspects, but making wisdom disappear is unforgivable. He set a path that led people in the region to forget and despise themselves, their cultures and civilisation, and become copies of others. A copy of the Mona Lisa is worth the price of the paper it is printed on. Similarly, the worth of a PhD holder in mathematics, who is a perfect copy of those who taught him in elite universities, is not worth more than the symbolism of the certificate awarded to them. Before I realised my mother’s mathematics, I was a copy of western academia, where my worth was connected to how many copies I was able to make of students I taught. The mind without wisdom as companion, can easily be deceived. Our minds today suffer from deceptions and illusions. A most important feature of wisdom is protecting learning in every child as an innate ability that is wiped out through grades and schools. To perceive oneself as a source of meaning that one can create through involvement in life, by reflecting on it and putting effort to independently investigate and form its meaning, is part of living wisely. This is made invisible. An act of wisdom is for each person to know one’s worth outside dictates of institutions, professionals, and trends.

The third phenomenon that followed the same logic was the concept of development as articulated by President Truman in his inaugural address (Jan 20, 1949) where he, without shame, described nations outside the US and western Europe as underdeveloped (degrading), and monopolizing the Euro-American tribe’s path as the only one to be followed, and that he is so kind to help countries around the world become developed (by becoming copies of the Euro-American tribe; I use tribe to stress it is not universal). The congruence of the underlying logic among Macaulay, Bliss, and Truman in subduing and controlling people is amazing and worrying. What is so worrying about it is that we embrace it and feel proud to be copies, perfect copies of those who themselves were subdued. The torn situation in our region today is a result of all that and totally invisible. We fight colonialism and imperialism in all aspects, except the invasion of our minds and perceptions.

There is no mathematics department that I know of that considers searching for the underlying logics in phenomena we live, worthy of being part of its concern. The problem is not in their non-concern but in our belief that those who step on our minds are superior morally and intellectually. There is a big difference between control and hegemony, which was clarified by Antonio Gramsci a century ago. Control is felt from outside, which needs to be resisted; hegemony is accepting control as progress. This was true in my relationship with the courses I mentioned at the beginning. I believed then that those who invaded my mind are superior; and most probably, students whose minds I invaded thought I am superior, a contaminating disease. One way to heal from all of this is to reclaim what was made invisible. My mother was a healer who helped me kick the Trojan Horses out of my system in several domains: in mathematics against mathematics' missionaries, and in religion against Christian missionaries. I happen to be a Christian Palestinian Arab who belongs to a family that most probably has been Christian since Jesus walked on the land of Palestine. For an American to come and preach to us about Jesus is simply rude. It is like me going to China to preach to them about Confucius! The situation we live in in our region prompts us to reclaim wisdom as manifested over centuries within the Arab Islamic civilization, starting with the 'House of Wisdom' in Baghdad that was established around 800 AD. Two characteristics of the Arabic language I find fascinating are: every word has multiple meanings (contrary to dominant words today that claim to have single universal meanings); and that the root of every word is a verb (the centrality of action in thinking). My mother did not follow the path from theory to application, but her knowledge was a mixture of her fingers, eyes, mind, imagination, the needle and the thread in her hands, the piece of cloth the woman brought, and the woman's body. Her knowledge was not linear. In addition, her knowledge had a use value; mine had an exchange value.

Pure sugar corrupts the body; pure mathematics corrupts the mind. That is why we need, more than anything else, to bring meaning back as a central concern in education and academia starting in grade one, and to pull out malicious words from text and academic books, just like pulling out sugar out of children's menus. This protects children from corruption at both levels: what enters our gut and what enter our minds. The difference between the two is that corruption of the body becomes visible while corruption of the mind remains invisible. Again, the mind without wisdom as companion can easily be deceived.

Another story worth mentioning here in relation to what is lacking in dominant ideologies is a survey that was conducted by the Christian Science Monitor in Boston in 1997 about which poet had sold most books in the US that year. The Monitor wrote: "In a country where Pulitzer Prize-winning poets often struggle to sell 10,000 books, Coleman Barks's translations of Rumi sold more than a quarter of a million copies." One comment made by an American was: "It's a matter of our enormous spiritual hunger".

If a mathematics teacher deals with a course (as I mentioned at the beginning) in a way within context and in harmony with wisdom, that would be wonderful. My quest is to stop teaching any subject without meaning, context, and action, not only in mathematics but in all subjects. The main values that govern formal education and academia are control and winning; and the main tools used include lifeless dogmatic language that claims universality as in text and academic books.

Another aspect, very much worth mentioning, is how Covid failed humanity and exposed schools and universities where all they cared about was covering curricula, giving tests, grades, certificates, collecting fees, ranking, number of prize-winning scholars and amount of grants in them. It also exposed enlightenment whose claims centred on the belief that if we focus on the mind, science, math, etc., we will be able to avoid widespread diseases. No school or university dared to get out of the track drawn for them.

My awareness of my mother's world was like a volcanic earthquake in my mind. It shook the foundation of what I acquired in schools and universities I had contact with, and at the same time produced a very rich lava at the intellectual level.

We need a vision, a compass, as a start, not goals. For me, the compass is the Arab Islamic Civilisation. I repeat, a main theme in this article is that there is no hope for humanity to survive on this planet without reclaiming wisdom in our daily living. One big obstacle to this lies in the fact that the dominant language used in official institutions is more like plastic where they do not change according to the place where they are. If one gets a degree in mathematics in any university, you can apply to any university in the world to continue to Master's study. The context is completely absent. Plastic knowledge, just like plastic flowers, remaining

the same, no matter where you take it.

A revealing story in this regard happened in the 1520s. William Tyndale wanted to translate the New Testament into English that people use and understand without the medium of institutions. King Henry VIII threatened to kill him if he did. Tyndale hid in Germany and translated the Gospel. The king searched for him, found him, and Tyndale was executed by being burned at the stake. What was the crime that Tyndale committed to deserve all that? Respecting people's ability to understand Jesus without institutions and clergy. We need to do the same in education: respect children's innate ability to learn without dictates by textbooks, licensed professionals, and vertical evaluation. All that they need is live real rich healthy soils of all kinds. They need institutions and professionals only to acquire technical skills and knowledge.

One quest I have is to reclaim what formal education and academia wiped out or made invisible. The title of the first article I published about the mathematics that my illiterate mother was practising, is: *Community Education: To Reclaim and Transform What Has Been Made Invisible* (*Harvard Educational Review,* Feb 1990). The best way to understand a phenomenon in life is not to look only at what it presents but also (and more so) at what it makes invisible.

Back to meaning, and to learning as an innate ability. *Jalal al-Din Rumi,* 750 years ago, wrote: "You have seen the picture but missed the meaning." How very true this is about today's world! We need to switch our focus from what is visible to what belongs to the roots. Rumi again: "Maybe you are searching among the branches for what only appears in the roots".

Reclaiming or creating multiple sites of learning [such as 'homes of wisdom' rather than being quarantined (bodily, mentally, emotionally) in school boxes over 12 years]. The idea that schools and universities are the only site of learning is fallacious. Prior to the rise of hegemonic universities, there were various sites of learning. UNITERRAS in Southern Mexico represent radically different and inspiring sites. There is much talk against colonial and imperial education that invades us from outside, but very little against Trojan Horses that invade and defeat us inside.

Simple ideas I used with small children included using a string and what they could do with it rather than the concept of straight line. Strings connect children to context, action, imagination, self-rule, and diversity in learning; in short, to be reflective thinkers at that young age. Starting with straight lines connect people to control, evaluation, judgement, rewards or punishments.

Social justice should be replaced by living wisely; equality should be replaced by every person is uniquely complete and cannot be compared with others along a vertical line; and human rights should be replaced by dignity. The difference between the first words and the second words is that the first are determined by official institutions and licensed professionals, while the second are the responsibility of people concerned. The upside-down world we are living in is incarnated by the fact that we consider a person who corrupts cars a criminal, to be punished, while one who corrupts nature is considered a scientist to be honoured and given medals!

My mother's unconscious revolt against the impudence of the mathematics I studied and taught; a revolt in silence, with dignity and which had gone deep within me and others who heard me tell her tale. She revolted not through words and concepts but in actions, lifestyle and with tools that are available to all people naturally: her hands, fingers, eyes, mind, imagination, needle and thread. The main message here is to remind us that we have, as human beings, so much within us that were made invisible but crucial in living wisely, in a world that keeps telling us we cannot learn and live without institutions and professionals. She violated what is universal, truths that claim to be singular and universal. She brought sanity and healing into my life. Some people asked, "What would you have been if you didn't go to universities?" Most probably I would have been a designer of clothes that are simple, beautiful, comfortable, and affordable …

Munir Fasheh was born in Jerusalem in 1941 and expelled with his family in 1948, just like hundreds of thousands from their homes. (There is a video by Ahmad Damin with Aljazeera, title The Red Stone, about the home he was born in and expelled from.) He lived in Ramallah most of the 76 years in exile.

Teaching mathematics to deaf children

Ben Bowen talks to Rosa Archer and shares strategies for teaching deaf children mathematics.

As the mathematics teacher, you are responsible for the mathematics education of all the pupils you teach. The pupils are all different, learn differently, react differently, come from different walks of life. I would like to talk about one of these groups: deaf children. 'Deaf' is actually a broad term that encompasses a wide and diverse spectrum. Some have mild hearing loss and manage well in mainstream schools. Some have cochlear implants to help them 'fit in' to their school life, yet are still segregated in a way, via their deaf units. Others, like me, are profoundly deaf.

The languages they use are also different. Some prefer to speak/lipread. Some like to sign to communicate. This includes but is not limited to the British Sign Language (BSL) and Sign Supported English (SSE). I have not included 'makaton' because this is the most misunderstood means of communication. Makaton is a communication system designed to support some pupils with SEND needs, particularly those with learning difficulties. Makaton does borrow signs from BSL, but it is completely different.

A brief history of deaf education

Before I discuss my experience of teaching as a deaf person, I would like to briefly summarise the history of deaf education. 1792 to 1860 was considered the golden age of deaf education through sign language. However, oralism was advocated by Heincke, who set up an oral school for deaf children in Germany in 1778. His ideology is that deaf children learn better via the use of lip-reading. This ideology clashed against the 'methodical signing', advocated by l'Epee who founded a school in 1760 in France. The conflict between the two ideologies continued for a while. This came to a head in Milan, in 1880. Here, at the second International Congress the Education of deaf children:

- 164 delegates attended from various nations. Only one delegate was deaf
- Speech was determined to be superior over any form of sign language
- Sign languages were therefore banned worldwide from educational practices
- Use of sign language declined for the next decade
- Many specialist deaf schools were closed down/adopted oralist approach.

The consequences were devastating for the deaf communities in the rest of the world. Now the oral method was considered the superior method of education, meaning those who teach deaf children using sign language were out of jobs, and stigmatised, having a severe impact on the development of sign language.

In 1978, Baroness Warnock published the Warnock report. This report recommended that all pupils with special educational needs (including deaf pupils) should, as far as possible, be educated in mainstream schools. So, increasingly, deaf pupils attended mainstream schools, rather than deaf specialist schools. Although this gained some support within deaf education, there was also significant opposition. There was also reaction from the wider deaf community who were concerned about the implications for the deaf community of the move away from schools for the deaf that were seen as central to the community.

Since then, the number of deaf specialist schools has dwindled. The most recent was the announcement of Hamilton Lodge School for Deaf Children that they will close down in August 2025 due to insufficient pupil numbers.

It is worth adding that, despite the technological advances over the recent years, the outcomes for deaf school leavers remain broadly behind their hearing peers and this has not changed. Deaf children can achieve academically **if** they are correctly placed and taught with their needs taken into account.

The challenges: what does the research show?

I want to move on to write about my experience of teaching deaf children while training to be a teacher at the University of Manchester. I will also relate my experience of teaching in mainstream. Teaching deaf pupils is an interesting challenge. If we agree that more than 80% of children's learning is incidental then deaf children are already at a severe disadvantage, as they cannot hear a conversation next to them, for example. So, being deprived of the main avenue for their learning, they must rely on the 20% that is being taught directly. Many fall behind in schools; on

average deaf learners are about two years behind their hearing peers by the time they leave school, especially in areas such as language comprehension, where hearing is vital. How are you supposed to learn to say the word and spell 'congruent' without hearing it first? This disparity in the learning processes leads to poorer working memory if the deaf child is not taught specifically to their needs..

Mathematics is heavily reliant on vocabulary. There are many field-specific words such as quadratic, reciprocal, trigonometry. As a teacher you will need to emphasise and re-emphasise the key words, at every opportunity. I incorporate this into my teaching by writing the key words alongside the learning objectives, and reviewing the words at every possible opportunity, including starters or plenaries. I ask learners to fingerspell the word or write the word on their mini-whiteboards.

A study by Conrad in 1979 showed that the median reading age of deaf children aged between 15 and 16.5 years old is 9.0. A similar study in the US similarly showed the reading age of 9.2. (Garrison, W. *et al*, 1997, p.81) This is why deaf specialist schools heavily emphasise setting time aside for reading time, to improve literacy. In my experience, every mathematics lesson with deaf pupils is also a language lesson.

The language and reading comprehension also pose further challenges in the exams; I noticed that deaf pupils struggle the most with the problem-solving questions that are heavily reliant on language, Take this question as an example:

> Ash is x years old.
> His brother is 2 years younger than him.
> His sister is 8 years older than him.
> The mean of their ages is 12.
>
> Form an equation and find out the value of x.

The deaf pupils must first read and understand the context, decode the information provided, and figure out what the question is asking for. They must also figure out what the steps are. This question is standard in formal assessments, but it is difficult for the deaf pupils. That is why in deaf specialist schools, the deaf pupils are allowed a BSL translation of the question (with restrictions, as some signs give away the meaning of keywords, such as mean). Therefore, in my lessons, I like to have a problem solving or sample examination question in my plenary. That way, learners have the opportunity to familiarise themselves with examination-style questions with these kinds of vocabularies, so they do not get overwhelmed when it comes to formal examinations. This approach is known as command vocabulary.

The advantages of teaching mathematics

Fortunately, mathematics can be a visual discipline. I utilised this key characteristic of mathematics as a tool to counter this language barrier, even for the concepts that are more abstract. For example: when working with ratios I will use the bar model. In pre-algebra, I will use prices associated with different items to represent values. When teaching fractions, I will use representations such as bar models or pie charts. There are so many models to use to show mathematics visually. The visual method benefits all learners. The Concrete-Pictorial-Abstract (CPA) theory of learning which is an applicable model of learning for deaf children. In my first placement, I learned and used the CPA approach extensively. For some classes, I would start at the pictorial stage. For others, I will start with the concrete stage. All pupils reacted positively to this approach. To teach fractions, I used pie charts for the pupils to fill in alongside the numbers to ensure they understood the concept and did not revert to their misconception that $\frac{1}{2} + \frac{1}{3} = \frac{2}{5}$. Once their foundational knowledge seemed more secure, I moved away from the pie charts. Interestingly, some pupils were able to write the fractions with no problem, while others would then draw their own pie charts and use them to solve the problem. I think this goes to illustrate that the pie charts really do make a difference to deaf pupils.

There is another challenge, poor working memory. Research has shown that working memory is correlated with hearing level. That is, the more someone hears, the better their working memory. Burkholder and Pisoni (2003) "found that children with cochlear implants who were able to speak faster also displayed longer memory spans, suggesting a relation between speaking rate and working memory [...]" due to "the early sensory, linguistic, and communicative experiences that these children were exposed to after receiving their cochlear implants." (p.68).

I believe this is due to being able to access more incidental learning if you are able to hear more. As a result, deaf children do tend to forget things more often, so mathematical topics need to be recapped often to allow them to retrieve the information better. Because of this, I would have to keep repeating the key words throughout, usually in the form of starters or plenaries, such as 'Define vertex'. otherwise deaf learners would forget the mathematical terms.

That said, I have observed that children who are exposed to signed language from an early age, particularly those from deaf families, do not follow the general trend as observed by Burkholder and Pisoni. This observation could be explained by Furth (1966) and Rapin (1986), "who suggested that deaf youngsters' poor results in reasoning tasks and educational assessments can be explained by an 'information deprivation' (Rapin, 1986, p. 214). Deaf youngsters lack access to many sources of information (e.g., radio, conversations around the dinner table), and their incidental learning may suffer from this lack of opportunity." (2002, pp.121). Those who are raised in a rich language environment that is accessible to them can pick up information visually instead, which may explain why my observations do not correspond with the findings by Burkholder and Pisoni. However, the English vocabulary does still need to be emphasised in my lessons, as English is considered to be deaf children's second language, after BSL.

The strategy of repeating key words, using both fingerspelling and in BSL signs, in my experience works well. This is then reinforced by another strategy that I learned in my second placement, which is in a mainstream school, but the concept I learned is invaluable, as it is beneficial to deaf children as well. It is called Realistic Mathematics Education (RME) (Freudenthal, 1973). The core concept behind RME is: we tend to remember what is relevant to our lives. For example, some of you probably learned French at school. So, since leaving school, how much French do you remember? For most of you, probably not much. The reason for this is, we do not use it in our daily lives. It is not relevant to our daily lives. So, we forget them to make room for things that are more relevant. As the saying goes, 'use it or lose it.' This is relevant to mathematics because, fortunately, mathematics has another key feature; its ubiquity. In other words, it is absolutely everywhere. The RME approach is used to put mathematics into the heart of daily living. With examples such as making a cup of milkshake, we use ratios. It puts mathematics into real life contexts and allows mathematics to be more relevant to young people's lives. So, once something is more relevant in pupils' lives, they are more liable to remember it. I would like to emphasise that the RME approach applies to all pupils, not just the deaf pupils. Thanks to the ubiquity of mathematics, I would argue that mathematics teachers have the most freedom on how to teach their subject. It is up to you how to use it. You know the pupils in your class the best. So, you can tailor it to your pupils. Pick the contexts that are most relevant to your pupils.

In my second placement, I took this concept a step further, by personalising the mathematics to learners' lives. For example, I modified an RME lesson to include specific popular games such as *Minecraft, Fortnite*, *FIFA 2024* and *Rocket League*, and asked the pupils what they preferred, then displayed it on the whiteboard in tally form. The pupils would then convert the data into pie charts. It was a wonderful lesson, as it was relevant and personalised to them. They would often make comments about the games, showing they were really invested into this.

Teaching in mainstream school as a deaf teacher

My second placement was very interesting as a deaf person; it was the first time going to a mainstream school as the only deaf person! I had two interpreters with me so they could rotate in my back-to-back lessons. It worked really well! I was able to delegate the control of sound levels to the interpreters. Also, with my interpreters present, the pupils are forced to behave better, as my interpreters are positioned at the back of the classroom. So, they are able to hear inappropriate conversations at the back of the classroom and inform me so I was able to respond. I have several examples of this happening. On one occasion a pupil was working on a laptop at the back of the classroom. Or, he was supposed to be, but my interpreter stood behind him and informed me he was watching a football match. So, I went to him, and he immediately shut down the window, but had to face consequences for his actions anyway.

I do have challenges, of course, with writing on the board and not speaking, so valuable learning time was lost. But the pupils seemed to adapt well to this, thanks to my interpreters relaying instructions. Another challenge I had was the interpreters' comprehension of mathematics. One interpreter was a good mathematician, so it was fine most of the time. However, I remember a specific lesson where this interpreter had to intervene as the other interpreter who was speaking did not understand me and started to create her own version of mathematics. This showed the importance of the interpreter asking me to clarify, rather than making it up, as it will reflect badly on me as a teacher. So, I feel it is vital to have an interpreter who is fluent at mathematics to be present in my mathematics lessons, as I did.

The issue with working in mainstream school as a deaf teacher, is the fact that the Access to Work Grant is capped at £64,000 per year. This is not enough to fund two full-time interpreters, assuming I will be able to find the interpreters willing to work for me pretty much full-time. That is the only issue! Everything else went smoothly. It shows that deaf people are able to teach in mainstream schools. Unfortunately, it is not

realistic at the moment, unless you are able to hear and speak well without an interpreter, which I cannot.

Finding my angle as a teacher to deaf children

When I began my third placement in a different deaf specialist school, I intended to incorporate both the CPA and RME concepts into my lessons. I had discovered that I struggled to teach the way I did in my second placement due to the smaller numbers of students in my class. However, the smaller number of pupils allowed me to really get to know them. As my placement was only six weeks long, I could not really utilise the personalisation until the final week. I realised that to personalise my lessons, I would need time to get to know my pupils. That is really important.

In my third placement, I had discovered the importance of the positioning of the deaf pupils. I was shocked to learn there is very little research on this, but it is important for the deaf pupils to have optimal positioning; at the front centre. When you teach other pupils, you can speak and write at the same time, and the pupils can copy down and listen to you. Deaf pupils cannot do that. They rely more on copying down from the screen/whiteboard, their eyes alternating between the screen and their workbook. This example also leads to 'concentration fatigue' - a common phenomenon among deaf people, due to their reliance on sign language and lip reading in order to understand what's being said.

To address this, they would also need some breaks, especially in double lessons. In my third placement, the seating plan was sort of U-shaped. This allowed me to teach the pupils more optimally as the pupils were all at the front row. However, the pupils at the edges were at a disadvantage as they were watching me at an angle.

Placement three also provided me with an insight in managing the behaviour of deaf pupils. Due to the U-shaped seating plan, the pupils could see each other at all times. This is an advantage, but also has its drawbacks as it also allows the pupils to sign to each other, often ignoring me when I am talking, prompting me to get their attention and then look at others to ensure they are looking at me, before continuing.

Going forward into my first year of teaching full time, I will also be teaching post sixteen mathematics at a deaf specialist school, as well as with the full range of younger learners. I am all too aware of another challenge; the terminology of A-level uses a language that is not part of everyday use, and mathematics teachers who teach using BSL need to get together to agree on the signs being used, and we are diversely spread across the country, making it difficult to do that. We do have a resource, the Scottish Sensory Centre (SSC) (see notes at end) , who have developed a glossary that covers some basic mathematics signs, but needs further development to include more complex terminology.

I found, in my preparation for becoming a full time teacher, that many signs are yet to be developed, or are too technical to be used to converse effectively in a lesson. So, I would need to create new signs to match the meaning of the words in post sixteen mathematics. It is a challenge that I am looking forward to meeting head on!

Ben Bowen is now a mathematics and science teacher at a deaf school.

Rosa Archer is the programme director for the PGCE secondary mathematics at the University of Manchester and had the privilege to be Ben's tutor.

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Notes

The glossary from the Scottish Sensory centre can be found at https://www.ssc.education.ed.ac.uk/BSL/index.html

Social justice, cognitive science and the powers of learners

Alf Coles reflects on our expectations of our learners.

The key question I want to ask in this article is: what do we think our learners are capable of? This question comes from a concern that messages from cognitive science permeating into schools may be sowing the idea that learning needs to be made simple. Inherent in the idea of making things simple is an assumption that learning is hard or, perhaps, that learners only barely possess the skills they need to learn. My worry about these assumptions is the implication for social justice; it may be the most disadvantaged students who stand to lose the most.

As I travel around schools in the south-west of England, I see an increasing number of examples of a lesson structure, or mathematics teaching approach, referred to as "I do, We do, You do" (broadly, the idea that the teacher models something with no student input, then works with students to do the same thing together, and then gets students doing the process independently). The origins of this idea are obscure to me – as far as I can tell it emanated from literacy studies – and is now linked to ideas from cognitive science and notions of cognitive load. During the "I do" part of a lesson, there is an idea that it is important there is no input from students, because it is easier on students' cognitive load if there is no distraction and if they can have a clear and uninterrupted model to follow. Learners and learning are assumed, on this model, to be fragile, needing careful handling and being easy to overload. There also seems to be an assumption that the learning path of a class can be planned out precisely.

I do recognise there can be advantages of an "I do, We do, You do" approach, in terms of teacher workload, and potentially in terms of managing behaviour. Part of my concern, however, is a worry that if we believe student learning is precarious and prone to relapse, these beliefs can become self-fulfilling. And, it seems likely that such self-fulfilling prophecies (about students' capacities) will most negatively affect those students already marginalised by class, race, gender, or socio-ecological conditions of life. In other words, unless we are careful, there may be negative social justice implications associated with the idea that the best way to start learning something new is to be silent and have it explained to us. So, is such a view of learners and learning really what cognitive science is telling us?

What does research say about cognitive load?

A key figure in research into cognitive load is Sweller, who has been researching in this area for over 30 years. While he has more recently engaged in pedagogical implications of his research, and endorsed ideas such as avoiding spitting learner's attention, I have found it instructive to return to some of his earliest formulations of the notion of cognitive load (i.e., I am not convinced his implications for teaching are warranted). In his earliest work, he makes use of the idea of a "schema" relating to what we are learning at any time (in later work, he makes distinctions between different kinds of knowledge and skills and does not refer to schemas). On my understanding, a schema might include knowing how to add fractions, knowing how to get to your nearest shop, knowing how to tie your shoelaces. Sweller (1994) argued that:

(a) We learn "schemas" (interpreted as "how we do something") and at first can only apply them with considerable effort. There are likely many items of information involved in a schema that need to be juggled or co-ordinated.

(b) Over time, if we become more familiar with a schema, then the process of automatization affects the schema itself. Many items of information become replaced with one item carrying all of the previous information.

(c) Through this process of chunking, limitations of working memory can be ameliorated, i.e., while there is a limit to the number of items we can consider simultaneously (the limits of our "cognitive load"), there is no limit to the amount of information that might be held.

So, if we believe these ideas, we can only attend to a limited number of items at the same time (which is surely right). And yet, the awarenesses contained in those items is unlimited. There is nothing, in this formulation, about learners needing to avoid distraction. Indeed, almost to the contrary, Sweller seems to be acknowledging that at the start of learning any schema, there are many items of information that need to be co-ordinated. So, actually, in Sweller's original research, far from viewing learners as weak, and learning as fragile, there is (to my reading) an acknowledgment of the strength of learners, in that we must have been able to juggle and co-ordinate

many items of information, in order to come to know the schemas that we use.

A metaphor of learning that seems common in England, is that of a journey (on a path), or a building (that needs solid foundations). My colleague Nathalie Sinclair and I have critiqued such images (see, Coles and Sinclair, 2022a, 2022b), concerned (again) about social justice implications. But a careful look at Sweller's original writing (above) indicates an image quite different from a building block model. If, to learn a schema, we need to juggle and co-ordinate many items of information, the image that comes to my mind is not of a clear, straight-forward learning path, that leads uninterrupted to a goal, but rather of a messy, complex, tangle, out of which, through effort, I discern some patterns.

What else does cognitive science tell us about learning?

There are of course many authors now writing about cognitive neuroscience and learning. To my reading one of the most profound is Stanislas Dehaene (2020). Through a review of evidence, Dehaene proposes there are "Four Pillars of Learning". These pillars are: (1) Attention; (2) Active Engagement; (3) Error Feedback; (4) Consolidation. I will focus here on the first two pillars, as these are the most relevant to the concerns of this article. In relation to attention, Dehaene writes:

> "A teacher's greatest talent consists of constantly channelling and capturing children's attention in order to properly guide them […] Attention acts as an amplifier and a selective filter […] Our attention is extremely limited, and despite all our good will, when our thoughts are focused on one object, other objects – however salient, amusing, or important - can completely elude us and remain invisible to our eyes. The intrinsic limits of our awareness lead us to overestimate what we and others can perceive […] When we teach, we tend to forget what it means to be ignorant." (2020, p.115).

This could be interpreted as an argument for practices such as "I do, We do, You do" (since our attention is limited). But there are several points to note. Remember, Sweller's research indicates there is a difference between the number of items we can attend to and the information those items can hold. And, our attention is limited to how many things we can attend to simultaneously, but not in relation to information. Looking closely at the quotation above, the key difficulty identified is how, as a teacher, I might work to *align* my attention with my students. One of the problems with modelling ("I do") is that, while presenting something, despite my best efforts, I may be attending to one thing and my students quite something else ("we tend to forget what it means to be ignorant"). Dehaene refers to several studies showing the importance of shared attention. What he goes on to argue is that a key to gathering attention amongst students (a precursor to being able to learn) is to generate *surprise*. I imagine that if mathematics lessons always follow the same structure and always begin with "I do", then there is a danger of it becoming increasingly difficult to generate the surprise that is needed to foster the shared attention that is necessary to learn.

In relation to active engagement (the second pillar), Dehaene is keen to distance this notion from discovery learning. What he does show, however, is that for learning to be effective, students need to have the opportunity to test out ideas, to voice half-formed predictions, to conjecture. A key to this pillar is to generate *curiosity* in students and give them frequent opportunities to put their learning into practice. This pillar speaks to the difference (often missed in psychology and pedagogy) between *recognising* and *producing*. An example (with thanks to Piers Messum) may be helpful here – I guess that most people reading this article would be able to recognise the Ford car logo. Please have a go now at drawing it [I suggest you pause your reading and try this].

If you found the drawing hard, and yet know you would know the logo if you saw it, you are experiencing the difference between recognising and producing. Another worry about a diet that always starts with teacher modelling is that students, in that phase, are in the role of recognising, not producing, mathematics.

What powers do learners have?

A key figure in the history of the ATM is Caleb Gattegno. Through an analysis of how young children learn (including how they learn their first language), Gattegno (2010) proposed that children have incredible powers of mind, or of learning, which they do not lose. His whole technique of teaching could be interpreted as an attempt to tap into these powers, which are: the power of extraction; the power to make transformations; handling abstractions; stressing and ignoring. One example of the use of these powers is when young children make mistakes such as "I runned". To make this error, children have noticed a pattern, generalised the pattern, applied the generalisation to a novel situation. In other words, Gattegno's point was that, in order to learn our first language, we needed to use powers of noticing, transforming, generalising, and they are still available for us, in learning mathematics.

So, putting together the strands of this article so far, early articulations of cognitive load theory do not imply

that learning needs to move from simple to complex, but rather recognise the need to co-ordinate a lot of information to learn something new. Evidence from cognitive neuroscience, for learning, emphasises the role of surprise and curiosity in learning. And, in contrast to constructing learning as difficult, and learners as fragile, we have the example of Gattegno, who points to the powers of learning of any child who speaks a language.

What might this mean in practice?

I began by articulating concerns I have around social justice, if we do not try to catch ourselves constructing learners as lacking in the skills they need to succeed. Gattegno implores us to consider the complexity of learning a first language, in relation to the relative triviality of what children are asked to learn at school. I want to close by offering three mechanisms that potentially could used within a (slightly loosely interpreted) "I do, We do, You do" structure, or within a scheme of work that is tightly constrained, and that could allow learners to access their powers of learning.

Using silence

I have written about silence as part of a collaboration with Laurinda Brown (Brown and Coles, 2008) where, for us, the teacher's silence was a way of drawing students into a task. The first use I made of this technique in a classroom was with the task "arithmogons".

With the figures pre-drawn, in silence, I write numbers in the circles (Figure 1a) and look around at the class. Then, slowly, and pointing at the relevant pair of circles, I fill in the squares one by one (Figures 1b, 1c).

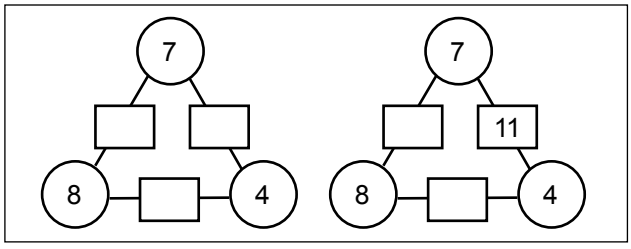

Figure 1a

Figure 1b

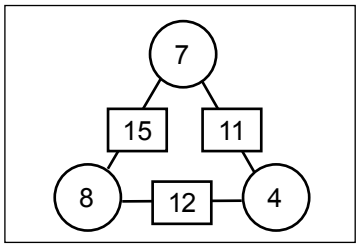

Figure 1c

I repeat for a second arithmogon (following the same process with different starting numbers in the circles). Depending on the group I am working with, I might do a third example. Having set up (I hope) how arithmogons work (without needing to explain anything), I then offer the challenge of Figure 2.

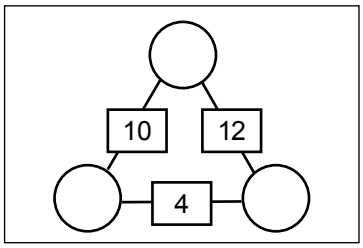

Figure 2: The challenge.

There can be a build of energy through the examples and a release into the challenge, which brings an excitement of knowing what the task is, and not knowing how to solve it (yet). The deliberate silence of the teacher puts the onus of sense-making on students. Having solved the problem in Figure 2, students can create their own challenges. Many questions are possible to pursue: Is there a method to find the solution? Is there always a solution? The task can be extended to arithmogons on a square or pentagon. For me, the task offers an example of what it might mean for students to *produce* mathematics. I do not mind if they find a general method, or not, as I know they will be doing lots of practice at addition and subtraction, and using powers of extraction and abstraction to notice patterns. In my experience, a teacher's deliberate silence, while engaging in some mathematics on the board, provokes curiosity and surprise.

Same/different

I associate the phrase "same/different" with Laurinda Brown's work and teaching (she developed the idea for her classroom in the 1970s and 80s). She and I also worked on this strategy in my classroom (Brown and Coles, 2008). For me, "same/different" became a key way of supporting students to make distinctions. I believe explanations in the classroom are inefficient and frequently ineffective. Laurinda and I planned lesson starts where there were two contrasting images. We would ask the students "what is the same; what is different?" confident in the knowledge that, if we have chosen the images well, what they will notice will be key mathematical features. Distinctions and concepts can then arise from what students tell me, avoiding the need for tortuous explanation. We are drawing on students' powers of stressing and ignoring.

For example, what is the same or different about the rectangles in Figure 3?

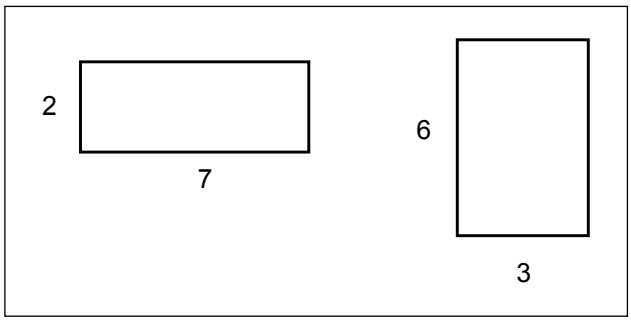

Figure 3: Two contrasting images.

At late primary school and beyond I am confident that someone in a class will notice the perimeter is the same and that the area is different. Someone may also notice that the area and perimeter have the same value in the second rectangle. Noticing that can lead into questions such as, are there other rectangles where area and perimeter have the same value? Again, there is an opportunity for producing mathematics. As with arithmogons, I do not actually care if students find other rectangles with this property – because I know that, whatever they are doing on this task, they will be working on the key distinction between area and perimeter.

Inverse operations

Following directly from Gattegno's idea of the power of making transformations, one straight-forward thing I can always do, for any topic, is to combine whatever process is being taught with its inverse. This idea permeates the NCETM's primary mastery professional development materials (see www.ncetm.org.uk). So, whenever I have to teach multiplication, I can teach division *at the same time*. Inverse processes gain meaning from each other. So, rather than having to learn two discrete and seemingly separated algorithms (such as long division and long multiplication) one thing I might try to do as a teacher, is seek a representation that allows for doing and undoing. And, this surely has to be possible for every process in school mathematics.

One example is the use of the grid method for both factorising and expanding expressions. To expand $(x + 2)(x + 5)$, I can use Figure 4a, filling in the squares by multiplying. (One task to set up how grids work, is to place four numbers on the outside, work out the four products inside, and the find the total of the products. Can you predict the total, from the four outside numbers?). For me the power of this representation is that if I have to factorise, e.g., $x^2 + 7x + 12$, I can use the same representation (Figure 4b). I know where the first and last terms have to go and so the question becomes how might the $7x$ be split. Students can explain to me how this has to work.

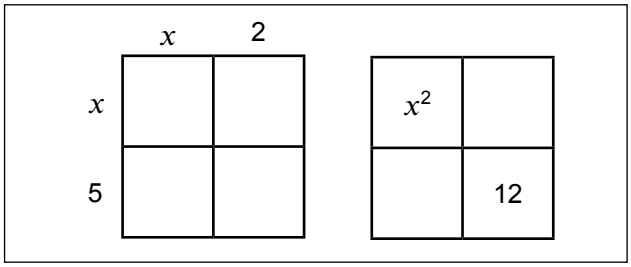

Figure 4a, 4b: The grid method for expanding and factorising expressions.

Discussion

I remain convinced that there is something engaging, as a learner, about being offered opportunities to produce mathematics, even for short periods of a lesson. What I also find is that it is often the most marginalised learners who value the opportunity to ask their own questions and having the space to notice (i.e., not always being in the position of trying to make sense of what others notice). The three mechanisms I have offered, above, might be seen as ways to dwell in a topic, or distinction, and to give time to allow complexity to settle, in a way that seems relatively well captured by Sweller's descriptions earlier in this article. And, although I have doubts about the overly individualistic view of learning that can arise from some cognitive science (and some of Sweller's own later developments of cognitive load theory), it is my firm belief that Sweller was on to something important when he proposed that *there is no limit to the amount of information that might be held in a learner's attention*. Now, there, it seems to me, is an inspiring idea to have in mind when planning, about each of my students' capacities …

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Maths at home for care-experienced children: The Letterbox Club

Rose Griffiths describes the growth of the Letterbox Club: a programme for children in public care.

'Social justice' is a large-scale, sometimes overwhelming aim. I have always wanted to teach mathematics effectively, so that people could better understand the world around them and become agents of positive change in their own lives and for others. I realized quite early on, like many readers of this journal, that children (and adults) learn much by exploring, solving problems, collaborating, adapting and creating. These are experiences where children can enjoy thinking about different ways of doing something and making decisions for themselves.

As a classroom teacher, I'd also be planning homework – tasks that I expected pupils to do, to support what I was doing in the classroom that week. But the children who most needed extra support were less likely to complete homework – so I started to think about what they wanted to do, rather than what I wanted. The opportunity to take this further came after a request from our local authority to look at ways of improving the chances of children in care.

Children who come into public care are amongst the most vulnerable children in our education system, and many fail to reach the levels of achievement that would have been possible if their lives had been more straightforward. The effects of trauma, loss and neglect are debilitating and things get worse in school if you find reading, or mathematics, frightening or baffling. It needs all the adults around the child to work together to help improve the situation, with school playing an important role, and home giving equally important but different kinds of support.

The central figure in this, though, is the child. How can we help them feel more hopeful, more interested, more confident in their own agency, and more likely to have a 'productive disposition' (Kilpatrick et al, 2001), so that they start to see mathematics as enjoyable, sensible, useful and worthwhile?

Last year, we celebrated the 20th anniversary of a small action research project for children in foster care, which aimed to engage children with reading and number games, and to support foster carers, adoptive parents and other family members who wanted to help the children do well. We started with just 20 children aged seven to 11 in 2003 in Leicester. The project has grown year by year, becoming a flagship programme for BookTrust, and we now reach over 13,000 children aged three to 13 and their families across the UK each year.

Mathematics by post: the Letterbox Club

Do you enjoy getting a parcel? Every child who is a member of the Letterbox Club gets a brightly-coloured parcel addressed to them, once a month for six months at a time, of books and number games to keep and use at home.

There are five levels of parcels, targeted at a suitable level of interest for children in five two-year age ranges. The age range does not appear anywhere on the child's parcels, which are differentiated by colour, so they can be used more flexibly if needed. Children who are in long-term care can be members of Letterbox Purple, then Orange, Red, Blue and Green; if they move home (sadly, still too frequent for many children in care), the local secure database of addresses should ensure that the parcels still reach them.

There are many practical aspects to sending a parcel, including the cost of the contents, the size of items that can be fitted into our easily-recognised, padded A4 envelopes and the cost of delivery. The central focus for our mathematics materials is number sense – counting, calculating and using familiar contexts, including money. That focus hasn't changed. Even so, any printed material can begin to look out of date (for example, in a shopping activity, DVDs are no longer something many people are likely to buy). There are also some games that do not work as well as we had hoped, so they need replacing.

Key attributes of a good mathematics activity

After a number of delays (including Covid), we have begun the process of reviewing all of our mathematics activities. Feedback from children, through questionnaires and postcards, gives us a good starting point, alongside comments from adults who have played the games with children.

The context in which the games will be played is very

important. As a former foster carer, I can tell you that after a day at school, children are often hungry, tired, perhaps a bit tetchy – and so am I. We wanted activities that didn't take too long, but that we could play again and again; where the instructions or rules were straightforward; and where we could alter the rules if we wanted to. Ideally, an activity would make us laugh and feel we had accomplished something. It should be suitable to be played with adults, or with children both younger and older than the child who received the parcel, depending on who that child chooses to share it with.

An extra consideration for children in care is that many will have arrangements for contact visits with birth family or siblings who don't live with them. We know many of our games have been useful on those visits, providing a few minutes of something to do together.

Looking at just a few items included across the five colours of parcels will give you a better picture of activities that we have found successful. In many cases, games are provided ready to be cut out, to keep costs low, but we know that many foster carers enjoy doing this with their children.

Different levels of challenge are provided, sometimes with two different games ('Starter' and 'Expert'). Alternatively, an activity can gradually become more challenging, for example by extending the range of numbers used – as in Magnetic Fishing, mentioned below.

What sort of things work well in each age range?

Counting in Letterbox Purple and Letterbox Orange (Usually ages 3 to 5 and 5 to 7)

Counting books are an obvious possibility, but learning to count cannot be done entirely through looking at pictures, and counting books at their worst can be boring, or be so busy and disorganized that it is impossible to count anything sensibly. But some stories are excellent. My favourite example for learning the string of numbers from one to ten is "*Where's Bear?*' by Emily Gravett. Bear and Hare are playing hide and seek, but Bear is hopeless at hiding. The story itself is engaging and the repeated counting out loud is purposeful, as we are giving Bear a chance to hide.

And, of course, counting out loud while one of you hides is a good follow-on activity. Counting backwards needs to become familiar, too, as in '*Mouse Count'* by Ellen Stoll Walsh, which features some clever mice who 'uncount' themselves as they escape from a hungry snake.

Figure 1: From '*Where's Bear?*

Counting objects happens in several of our activities, especially for younger children. Each parcel in Letterbox Purple includes a different animal finger puppet, with a 'collector's card' in the first parcel showing all of them, with boxes to tick as each one arrives. Over the six months, children accumulate these little animals, which they are encouraged to count, play with, hide and find. Each month, there is one more to count!

In Letterbox Orange, Magnetic Fishing (made with paper clips on cut-out card fish) provides enough fish to count to 6, then 12, then for adding different piles of fish for larger totals if wished. A set of maracas in Parcel Six gives the chance to count sounds not objects, and to practise keeping in time when singing or dancing.

Familiar games in different versions

Games such as Bingo, Pairs, Dominoes and simple track games are very useful, as most adults and many children will have some idea of how to play them already.

With Bingo, the child can be the Caller, or a Player, or both – there is mathematical activity within both roles.

The game of Pairs, in commercial sets of cards, can be very frustrating – it's not uncommon to have a set of 36 playing cards, with 18 different pairs – making the odds of finding a match when you turn over two cards quite small. To give a better chance of success, our set has 30 cards, with five groups of six cards that match; this encourages children to remember the number shown on a card they turn back face down, and its position, so they can begin to be even more successful at finding a pair.

A set of traditional Dominoes helps children learn the familiar dot patterns also seen on many dice. They can be used in many ways, for playing a game or for building. Having doubles and blanks (with zero dots) extends the range of possible games you can make up, including ones where you just add totals of dots when you turn over two, three or four dominoes at a time.

Saying, reading and writing numbers in Letterbox Orange and Blue (Usually ages 5 to 7 and 7 to 9)

Within each age range, we aim to provide a sensible progression of activities across the six months. For example:

- Parcel One in Orange has Magnetic Fishing, counting fish, hearing and saying the number names as you count out loud.
- Parcel Two links the spoken number with the number symbol: counting the number of animals in a group, and matching that to the printed number, in the game of Pairs.
- Parcel Three takes this a step further: reading the number names written as words, and finding the symbol to match, for Bingo. Then children practise writing numbers themselves, with guidance about making them clear for other people to read.

Calculators, addition, subtraction, multiples in Letterbox Blue and Red. (Usually ages 7 to 9 and 9 to 11)

A calculator is included in an early parcel for Letterbox Blue, partly to provide a method of checking answers for a game called 'Add or take away', but also because children can then use it to experiment with calculations of their own. Even though many people have a calculator on their phone, having a basic calculator of your own still seems popular with children, so we may produce a little booklet of suggestions for solo challenges and games you can play with a friend. As ATM members know, there are many!

Figure 2: Add or take away.

Our focus across this age range is firstly on addition and subtraction bonds and mental arithmetic, and on increasing children's familiarity with multiples of 2, 3, 4, 5 and 10. Very few of our activities involve writing; we try to provide different ways of thinking about the links between numbers, and how you can build on what you know already.

In the last parcel for Blue, there is a track game at two levels: the £20 game, where you win pretend £2 coins if you land on squares showing £2, £4, £6, or £8, until you have £20. The expert game, the £50 game, uses pretend £5 notes to concentrate on multiples of 5 to 50, with amounts like £5, £10, £15 on the winning squares. Some foster carers have reported that their children have tried to use £2 coins *and* £5 notes to play the £20 game – and they have realized that they can't always make the amount they need with fivers, but for £10 or £20 they can. A useful thing to find out for yourself.

Figure 3: The £100 game.

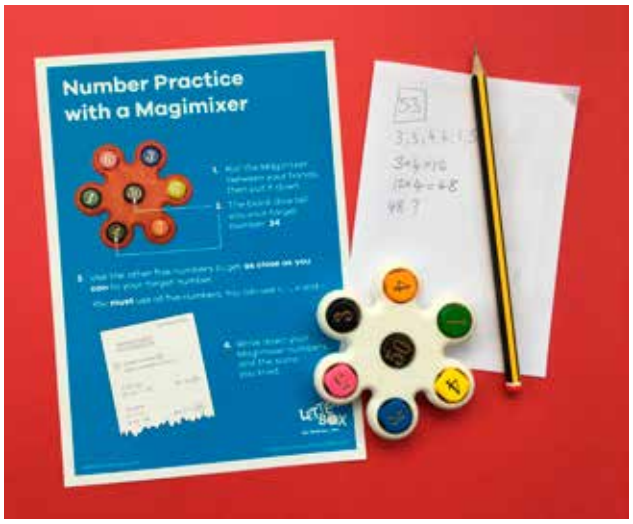

Figure 4: Magimixer activity.

Figure 5: Letterbox Club Times Table postcard.

A similar track game in Letterbox Red concentrates on grouping in tens until you have one hundred: the £100 game. Using just £1 coins soon becomes tedious when you are counting up to £100; being able to change ten £1s into a £10 note makes life easier, and helps make sense of place value.

Like many of our games, the £100 game provides equipment that children often use for other games of their own. There are many children who set up their own role-play shops; or as one ten year old girl told us, 'I just take all the money to school in my pocket, then I can pretend I'm rich!'

Working with larger numbers in Letterbox Green (Usually ages 11 to 13)

By the beginning of secondary school, the attainment range for mathematics has spread considerably and children's interests are even more varied, so choosing books and activities for this age range is even more of a challenge. There is still a need for encouragement for children who are finding maths difficult, but activities must not seem childish.

If a child has been a Letterbox Club member at a younger age, we know that many will still be using activities from previous parcels. New versions of earlier games can still be very welcome, such as 'Double your Money', a variation of the £100 game. We have also made a leaflet of games using a pack of playing cards, and another using our own pack of cards showing numbers 1 to 100.

Possibly the most successful activity has been the Magimixer, a set of seven dice held in a frame that you can roll between your hands. It's like Countdown: you have a target number and have to find a way of making that total (or something close!) using the five other numbers. Of course, you could use number cards instead – but the gadget is tactile and satisfying.

Extending Letterbox Club's reach

Letterbox Club's parcels were initially compiled for children in care, but they are now also used for many other children in vulnerable circumstances, including those who have been adopted, have an allocated social worker, or are young carers themselves. Most of Letterbox Club's members have their subscriptions paid from Pupil Premium or Pupil Premium Plus, often through the local Virtual School. There is also an increasing number of schools using this programme as a way of providing extra support to a wider range of pupils and their families for reading and mathematics.

If you would like to know more, go to the BookTrust website and search for Letterbox Club.

Further reading

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Embedding mathematics for social justice in a primary school

Joel Kelly, Caroline Hilton and Pete Wright explore how primary schools can bring in and benefit from Social Justice Mathematics following their recent project in West London.

Context

In 2021, we began the 'Primary mathematics and social justice project', which introduced social justice mathematics in a range of primary classrooms. We wanted to explore how primary school teachers could maintain and build upon their initial interest in addressing social justice issues through their teaching of mathematics and to consider how to help students develop their critical understanding of mathematics as well as their collective mathematical agency. Our use of social justice mathematics includes sociomathematical agency, which we define as the ability to use mathematics effectively to argue collectively for social change (Kelly et al., 2023).

The benefit of social justice mathematics in the classroom is supported by a raft of research (See, for example, Gutstein, 2006; Wright et al., 2024). We believe educators should open up real-life situations to examine the underlying ethical issues that we face as a society. The importance of students solving social issues and inequality and becoming active citizens has been argued by Skovmose (2011).

Our project aims:

1. To allow children to see maths in a different way.
2. To increase teacher input towards planning tailored maths lessons involving social justice.
3. To increase children's understanding of how maths is used in their world and to see maths as a subject that does not simply start and end in the classroom.
4. To increase the pupil's sociomathematical agency.

A model of participatory action research aimed at building and maintaining teachers' agency and self-efficacy in developing practice was adopted. Our research team consisted of three lead researchers (the authors) and six teacher researchers. The project took place in two London primary schools.

There were a series of research team meetings, facilitated by the lead researchers, to collaborate and explore ideas. Two cycles of participatory action research took place. They involved an initial meeting focussed on planning, ending with a session in which the lessons were evaluated. Participatory action research lent itself well to this project as it allowed a lot of room for self-reflection and development.

Interviews with teacher researchers at start and end of the project provided a wealth of data. Themes were drawn from the data and explored. There was a consensus that the social justice element of the lesson improved engagement:

I found all the children were really engaged and were really passionate about putting forward why their idea was the strongest. (David, Year 2 teacher)

I think, by presenting maths in a different way, I've had less of a "I don't want to do this, this is boring" reaction, and then seeing "actually, this is maths used in a different context". (Rose, Year 5 teacher)

Other key themes included:

1. Students' growing appreciation of how mathematics can be used to argue collectively for change. Teachers noticed that students entered mathematical discussions passionately and enthusiastically using maths to support their thoughts in the lessons.
2. Teachers also noted the benefits of reflecting and gaining professional development through this project. They shared the benefits of working collaboratively with other teachers to plan the sessions. The interviews echoed a clear, positive message of teachers broadening their perspectives on practice. Following this, some of the teachers began to incorporate aspects of social justice in their mathematics lessons outside of this project even when using schemes.
3. Teachers noticed that some of their more reluctant learners were more engaged in these sessions. They shared that they felt the learning environment was more inclusive.

Embedding

Social Justice Maths Days

After the research project concluded, there was a conversation around how best to continue the positive work that had begun. Working both as a member of the senior leadership team and in the field of research, I (Joel, first author) found myself in a unique position in which I could facilitate the implementation of academic research in my own setting, often including neighbouring schools in our endeavours. As the project only included a few teachers in my school, I wanted to give all teachers the chance to have a go at planning and delivering social justice mathematics to their classes. At this juncture, lacking time and the necessary resources, it was not possible to create a new mathematics curriculum including social justice elements throughout.

Furthermore, when planning lessons or bringing in new initiatives, one of the largest hurdles any teacher faces is finding space within the already packed curriculum. The primary mathematics and social justice project had been successful and was spoken highly of by the teachers who took part, but the question remained how could we do this consistently moving forward?

In my leadership role I was able to put forward a solution that would not increase pressure on the curriculum whilst allowing space for social justice mathematics to take place. I initially put forward a plan to do a termly Social Justice Maths Day across all of the year groups in the primary phase. The guidelines were simple: take all of your mathematics lessons from the chosen week and arrange to do them one after the other on the allotted social justice mathematics day; keep the learning intentions, coverage and skills but change the tasks to include a social justice element; stitch an overarching social justice theme to flow through the lessons (for example, lowering pollution in the local area). Here the teachers could decide on a social justice topic or ask the children what they thought was fair or unfair in their world.

The social justice day might fall, for some year groups, on a 'coordinates' week for example. This gave teachers licence to get creative and resulted in some of the most engaging mathematics lessons I have seen the children take part in (see examples below).

I created a teacher guidance document referring to the principles, aims and outcomes of the sessions as well as where to seek support from colleagues. Keeping an open dialogue about the project was key to nurturing creativity. After circulating it I used a staff meeting to go through the concept as a group. This was a great opportunity for the teachers involved in the research to lead, sharing their own examples, thought processes and their enthusiasm for the project.

Mirroring the project I asked teachers to plan in year group pairs to create their lessons. This sparked talk, excitement and creativity. Planning and resourcing took place in the teachers' planning time, with the partner teachers working collaboratively. The lessons were planned with their particular pupils' interests and needs in mind, with a level of personal tailoring that a commercially published scheme simply does not have the capacity to achieve.

This project involved teachers, early career teachers and student teachers and has been met with enthusiasm from the educators involved. The consensus was that the teachers observed their class seeing mathematics in a new way as well as seeing mathematics as a tool for change in their immediate environment. They saw the lessons as more inclusive as they allowed pupils who may usually say they 'don't like maths/can't do maths' to see the subject in a new light. Further to this, the teachers noted that helping the children to develop their sociomathematical agency has had a positive impact on how the pupils see mathematics as a tool for change.

Sustaining the Project

With the best will in the world, it is hard to maintain new initiatives in a primary school. The growing workload in schools often means that new projects that educators are passionate about often get put to the side when contending with all of the demands of the busy school year. I have found that if a project needs to be implemented then it also needs to be protected. I included social justice mathematics in our school improvement plan under the heading of 'the school centre for excellence'. This meant that we, as a school, could quality assure its rollout and had to track and rate its progress. Once something is in the school improvement plan, it will be prioritised and have the best chance of going ahead.

Furthermore, other simple steps include setting clear days each term for the social justice mathematics days in the school calendar and creating banks of examples. We began by sharing examples that came

Figure 1: Mapping a village.

up during the initial projects. These were shared by the early career teachers who came up with them, putting those staff members in leadership positions. Our school celebrates our social justice mathematics days in our school newsletter and shares the successes of the project with staff, parents and governors.

Examples

Since the start of this project the teachers at The Blue School have produced their own brilliant ideas for social justice maths lessons and projects. Below are just a few of the activities that we have enjoyed so far:

Reception & Nursery (ages 3-5)
The children role-played, in pairs, buying an item. Afterwards they were asked how much the person who bought it should have and how much the friend should have. The children practised halving, sharing and reasoning.

Years 1 & 2 (ages 5-7)

Voting for a story. The children were given four stories to choose from. They then collected data from the class on which was the most popular and recorded it on a bar chart. Afterwards they discussed the most and least popular. The phase then split into four so that each child could go and hear the story they had picked. They could physically see how many people voted for their choice in their groups.

Debate the pros and cons of a four day school week. In groups, the children are encouraged to use numbers and time to support their arguments. At the end the arguments are presented, the class votes and the data are recorded in a tally chart.

Years 3 & 4 (ages 7-9)
Design a multi-faith worship village and map the coordinates on an Ordnance Survey Map. Pupils considered fairness, shape, space, quantity, access and community whilst accurately plotting a village.

Collect and explore the data around responsibilities and the rights of children using Venn diagrams, tables and charts.

Using the book ‘If the World were a Village’ (Smith, 2002) as a starting point, the children considered how the world’s resources are split. They then came up with their own 100 people world village and considered how to make the division of resources fairer.

Map out the playground. The children collected qualitative and quantitative data on how accessible the playground was for all children both now and in the future. They explored the use of space and considered how accessible and fair it was. After they used their findings to redesign the playground, using costings from accessible playground equipment brochures. Finally they presented their plans and reasons behind their decisions. (See Figure 1)

Years 5 & 6 (ages 9-11)
World record temperatures. The children discussed global warming, ordered record temperatures, used ‘less than’ (<) and ‘greater than’ (>) symbols to compare the temperatures and rounded up to the nearest whole number.

Debate the length of lunchtime and the subsequent impact on learning time. Pupils came up with data around timings. They created their own timetables for the day. They voted on which was the best option using a tally to record the data.

As you can see there is some overlap of content in the examples above. This comes from teachers sharing ideas as well as natural themes arising when

we ask the children how we can make their world a fairer place. The teachers' planning has become more and more adventurous as time has gone on and I find myself in awe of the creativity of our incredibly talented staff.

Further to working with educators, I have also worked with a group of Year 6 Leaders on this project. I invited them to help run social justice maths tasks for their fellow students at lunchtime as part of the social justice mathematics day. They were excited by the project, which involved them having their own outdoor area set up and lots of stickers and house points to give out. We worked together to come up with three questions, with the aim of all children being able to accomplish at least one:

1. Three teddies want to share this pizza. How many slices do they each receive?
2. Two friends share a pizza. How many pizzas do they need to buy to share the slices equally?
3. How can you share a pizza, with nine slices, fairly between four friends?

This lunchtime activity was very popular, with the majority of the school having a go (5-11 year olds). The children were intrigued by the opportunity for pupil-led tasks and the excitement from succeeding and being rewarded with a sticker/house points had a ripple effect across the school playground.

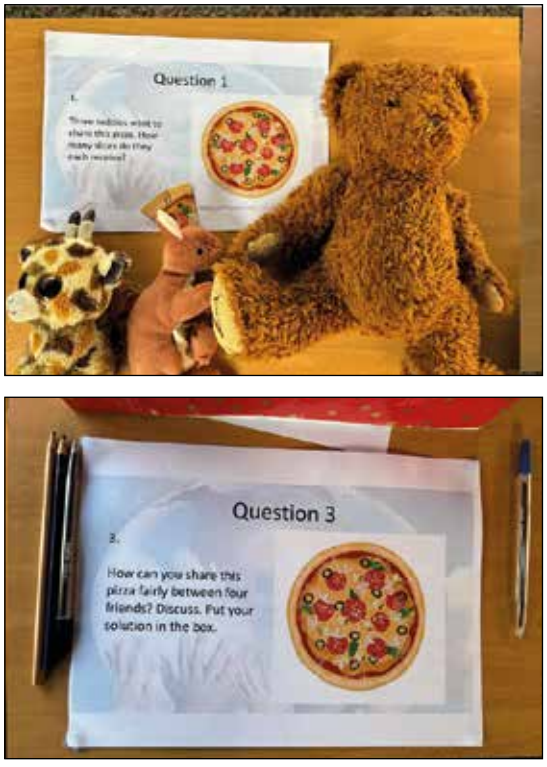

Figure 2: Sharing pizza.

It is worth noting that we always try to make the third question one that encourages problem solving skills, lateral thinking and social justice. It is also the question that we sift through the answers of and select three winners from. During this process I worked with my Year 6 Leaders and watched them sort the answers into piles. Out of the successful answers, they grouped into three brackets. In third place were answers that said: 'Give everyone two slices each and throw one away/leave one'. In second place they put all of the answers that said: 'Everyone gets two slices each, then cut the last piece into quarters'. For first place the Year 6's picked an answer that along with a clear diagram said: 'Just cut the pizza into four equal pieces'. We were all impressed that the pupil had been able to take away a bit of information (9 slices) to make solving the problem simpler.

Our social justice competition winners are presented with awards during our celebration assembly. Our Year 6's invite the children with the top solutions to the front of assembly and take time to read out their solutions and methods for others to learn from. I have seen children use similar solutions for different problems afterwards and have found the project empowering for the Year 6 Leaders and the children across the school.

I noticed a positive change in the school's culture toward social justice. One example of this was when the Year 6 Leaders began bringing their own ideas for social justice questions based on change that they would like to enact. Recently, two of the leaders posed the question, "A Year 6's mathematics lesson is 1 hour long. Their English lesson is 45 minutes long. How can we make this fairer? Please write out your answer and reasons behind it."

The top answers to this problem were offered to SLT by the children for consideration when timetabling the following year. This was a positive message as the Year 6's wanted to leave behind a fairer legacy even though it would not impact them directly, having left for secondary school.

Conclusion

Teachers are creative planners. Schemes can be useful but they stifle creativity and the opportunity to tailor maths to the children's world, both present and future. A balance of scheme coverage and the opportunity to engage with social justice elements of maths has been seen to raise the engagement and agency of both teachers and pupils at our school. A teacher shared with me: "Children who like maths will

still do so; those who do not think it is for them have the opportunity to see it in another way". Changing the tasks within a scheme or previous planning is a straightforward way to make a start on including more social justice mathematics within the curriculum.

Giving children the tools to tackle problems that their generation will face in the future as well as opportunities to improve their current environment, engages both students and teachers alike. The social justice mathematics days have been a success and have allowed teachers and pupils the chance to see maths in a different way. I have noticed an increase in children using mathematics to solve problems outside of the classroom, whether it is petitions to make their world fairer or timetabled suggestions of popular playground areas. Children are applying their mathematical knowledge and skills to enact change. Furthermore, it improves mathematical language as it involves lots of discussion; I have seen the youngest children understanding and talking about equality and equity.

For this project to be successful, it is important that there is clear guidance for staff as well as scheduled time to plan, do and review booked into the school calendar. An ideal way to protect the new initiative is to talk to SLT about adding it to the school improvement plan. This way the initiative can be prioritised, quality assured and the benefits to both educators and pupils can be realised.

Figure 3: Allocating play fairly.

Our journey is ongoing and I look forward to finding new and exciting ways to involve social justice mathematics in our day to day curriculum. There will continue to be natural growth as we provide teachers with more licence for creativity. I will also look to share our practice and collaborate with other schools and teacher training settings to develop ideas further.

Find out more about the project at:
https://mathsocialjustice.org/research

Joel Kelly is a member of SLT at The Blue School C of E.

Dr Caroline Hilton is an associate professor at the UCL Institute of Education.

Dr Pete Wright is a senior lecturer at The University of Dundee and convenes the Teaching Mathematics for Social Justice Network.

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Teaching mathematics for social justice: Meaningful projects for secondary classrooms

These activities are taken from the ATM book, *Teaching mathematics for social justice* by Pete Wright.

HOW FAIR IS FAIRTRADE? *FAIRTRADE CHOCOLATE 1*

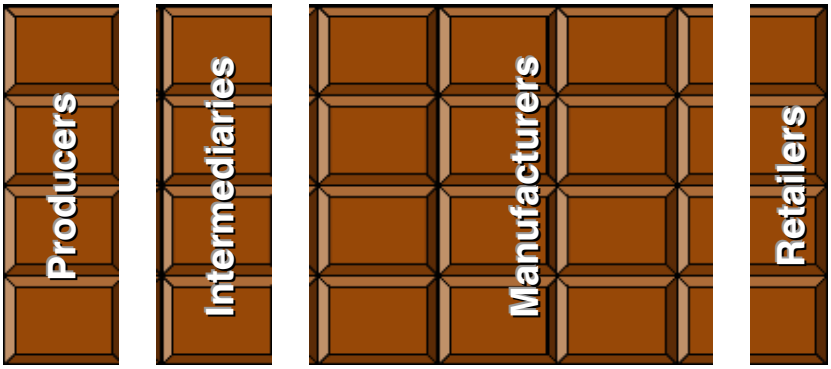
Most chocolate is consumed in rich countries in Europe (50% of world sales) and the US (22%). In contrast, cocoa beans are grown in tropical regions near the equator where many farmers and workers who produce them earn less than £1 per day.

By far the largest share of the revenue from the cocoa in an average chocolate bar goes to the manufacturers and retailers (based in the Global North).
This imbalance has increased in recent years as shown in the table below:

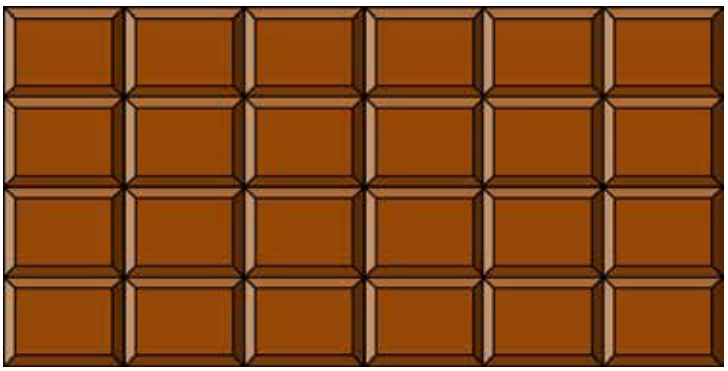
	Share of revenue from the sale of an average chocolate bar	
	… in the late 1980's	… in 2012
Producers	16%	6%
Intermediaries	16%	7%
Manufacturers	56%	70%
Retailers	12%	17%

(Source: Make Chocolate Fair! Campaign)

The distribution of revenue from chocolate bars in the late 1980's is displayed below:



Display the distribution of revenue from chocolate bars in 2012 in a similar way.



- Compare the two diagrams.
- What do they show you about revenue from chocolate sales?
- How else could you display the data?

ATTITUDES SURVEY

FAIRTRADE CHOCOLATE 3

Carry out a survey of the attitudes of people who regularly purchase chocolate towards buying Fairtrade products.

Think about the question that you want to explore. You could compare attitudes between males and females, between different ages or between other groups.

◆ Design a data collection sheet to record your results.

You could adapt the following:

Data collection sheet

To what extent do you agree with the following statement?

'I would be willing to buy Fairtrade chocolate rather than conventional chocolate'.

- ☐ Strongly agree
- ☐ Agree
- ☐ Neither agree nor disagree
- ☐ Disagree
- ☐ Strongly disagree

How much more would you be willing to pay for a small bar of Fairtrade chocolate?

- ☐ Nothing
- ☐ Between 1p and 10p
- ☐ Between 11p and 20p
- ☐ Between 21p and 30p
- ☐ More than 30p

Are you: Male ☐ Female ☐ Age: _____

◆ Transfer your results for each group to a frequency table similar to this one:

Frequency table for females

Response	Tally	Frequency
Strongly agree		
Agree		
Neither agree nor disagree		
Disagree		
Strongly disagree		

◆ Compare the frequency tables for different groups. Write about what you notice.

◆ Analyse your data and present your findings using statistical diagrams.

Hints: You could …

– draw bar charts, pie charts, pictograms, frequency polygons or box plots.

– calculate the mean, median, mode, range, etc. for each group surveyed.

◆ Write a conclusion comparing different attitudes towards Fairtrade.

Teaching mathematics for social justice is available from the ATM at https://www.atm.org.uk/shop/TeachingMathematics-for-Social-Justice-Book-Download-/act099pk Price £16.00 for non-members and £12.00 for members

Piecing together mathematics teaching and decolonisation

Tazreen Kassim-Lowe offers a personal perspective on decolonisation and mathematics teaching.

A patchwork perspective

As a teacher, one October, I refused to organise black history month claiming that *'every day is black history month in my classroom'*. This was a bold statement looking back. To be clear, I fully support black history month; however, I do question its tokenistic nature in some cases. What I meant was, when curating my curriculum, I would not wait for a specific month to showcase the creativity and pre-colonial narratives of historically minoritized people. I rather (attempted to) weave these narratives into the curriculum meaningfully in my own primary school classroom and did not consider them an afterthought. My bookshelves were stocked with books like 'Africa is Not a country', my history curriculum was non-Eurocentric and, when I taught poetry, I always started with poets from alternative language traditions and cultures. However, teaching mathematics always seemed separate to these efforts.

Here, I offer an insight into what it is like to be a British woman of mixed heritage, who has, almost always, operated in majority white spaces in education. Lately, I have been pondering the complexity of social justice in mathematics teaching and in particular what decolonising the mathematics curriculum looks like in British classrooms – *is it possible to decolonize a curriculum in a country that has historically been the coloniser?* In this article, I offer my thoughts on the complex relationship between mathematics and decolonisation. Secondly, I discuss decolonisation in relation to my teaching and unpick my own practice. Finally, I discuss a quilting activity that has the potential to *rehumanise* mathematics and how engagement in this activity has made me reflect on the relationship between the social and the mathematical.

It's not black and white

Colonisation is the systematic dehumanisation and oppression of large groups of individuals (often racially and geographically motivated) and includes forced assimilation and doctrines of inferiority. Although Britain's history of colonisation may seem like a faraway thought, the hierarchical values it legitimised still permeate the education system in Britain which often reproduces societal inequalities. This is despite the best efforts of teachers who aware of social justice issues. If colonisation is about building power and *dehumanisation,* I believe *decolonisation* is about building community, belonging and r*ehumanisation.* It is easy to think of mathematics as separate to colonialism. However, mathematics is not apolitical and has been used as a tool to legitimise colonial acts such as slavery; for example, calculating the capacity of a slave ship and the weight of bodies as 'cargo'. So, if mathematics has been used as a tool to dehumanise, as a teacher of mathematics, how do I rehumanise mathematics teaching? As I reflect on my class teaching practice, I wonder if I was successful in this endeavour and what I might do differently now as a more experienced teacher and researcher of mathematics.

Unpicking my own teaching practice

On reflection of my time class teaching mathematics in British classrooms, I always found myself struggling to find opportunities to meaningfully confront colonialist ideology, which legitimises the exclusion and hierarchical oppression of certain, often racialised, groups. Hypercritical of myself, I felt as though I had a duty to rehumanise mathematics. Yet, nothing I did seemed to do be enough. If I could communicate with my past self, I would remind myself that I am one person trying to find a way *towards* decolonising the mathematics curriculum and to be kinder myself. Below are some specific examples of where I have questioned, *critiqued* and given myself clemency for past decisions.

Culturally sensitive or culturally clumsy?

Was it enough to change the names of children in word problem from Alex to Amir? *Not really. Seems a bit tokenistic in hindsight. But at least I was aware of the demographics of my class and attempted to respond to this year on year.*

Data criticality or tangentiality?

Was it enough to remind children about 'ethnic lumping' when we discussed statistics? Or to criticise data representations and remind them data is only one side of the story? *Not on its own, no. Seems a bit tangential.* But at least you made time to have these

conversations in a very busy curriculum timetable.

Meaningfully or superficially putting oneself in the mathematics?
Was it enough literally to insert myself into the context of the mathematics? What about the photo I used of my sisters and me? If Miss Kassim-Lowe is x cm tall and Sister 1 is y cm shorter, and Sister 2 is half as much as Sister 1 plus 50cm. How tall are Miss Kassim-Lowe's sisters? I asked the children to go home, measure family members and come back with similar problems. I*s that really 'putting oneself' in the mathematics? Or is it a little superficial?* Look, the children enjoyed that activity. Some of them even brought in photos and from this we had beautiful conversations about who we are.

Relevant or irrelevant contexts?
Was it enough to contextualise mathematics in a way I assumed to be meaningful? When comparing and ordering numbers up to 1,000,000 I used population counts of countries and towns that my pupils had a link with. *Do 10-year-olds really care about populations? I doubt it affected how they accessed the mathematics.* I think the children felt seen and that I had listened when they spoke about their families and their identities.

Mixed experience groups or all mixed up?
Was it enough to never use ability groups in mathematics and rather place children together based on what they can learn from each other's ways of working and thinking mathematically? *Ability grouping would have been a lot easier to manage. A part of me still thought of them as 'numbers' related to a test or level.* At least I tried to reject deficit narratives and hierarchies that leave some children feeling like mathematics is not for them. I think everyone felt as though they could learn something from one another and that is worth acknowledging.

By looking *inwards* on my past attempts to rehumanise the mathematics curriculum, I realise that there could be multiple 'ways in' to decolonising the curriculum, but the intent can quickly become superficial and tokenistic. Years on, my role as a researcher has allowed me to look *outwards* at how others navigate connecting decolonisation and the teaching and learning of mathematics.

Connecting the pieces

I approach the topic of rehumanising mathematics as a decolonial act tentatively. I still have much to learn about what this means in a British context. Jodie Hunter's (2022) work with Pacific islanders' everyday mathematics seemed, to me, to be all about decolonising mathematics through rehumanising. Jodie gave children on Pacific islands a camera to record their mathematics and then used the images to talk about their indigenous ways of knowing the world through mathematics. Although still extremely complex, decolonisation in places that have historically been colonised, seems slightly more tangible.

Whilst grappling with my own understanding of decolonising the mathematics curriculum in the British context, I found myself returning to my memories of teaching in Zimbabwe where decolonising the curriculum seemed to be high on the agenda; Ndebele, a heritage language, was a compulsory part of the curriculum; oral history traditions were a valid form of communication and music, chanting, movement and dancing were a part of everyday learning. In my return to these memories and further reading, I have re-discovered the philosophy of '*Hunhu'*. A Shona word that means character/trait/person/being, *Hunhu* is a philosophy that aims to rehumanise education (Hapanyengwi-Chemhuru & Makuvaza, 2014). I feel that rediscovering '*Hunhu'* has validated my 'way in' to decolonising the curriculum. In the next section, I outline how merging the human-centered nature of this philosophy with quilting as an anti-colonial act, has the potential to connect individuals to the mathematics, each other and the world.

Hunhu Quilting

Quilting is simultaneously social and mathematical. According to Ptolemey et al (2023), quilting can act as an anti-colonial tool that expresses one's story, identity and relationship with others. I believe that quilting has the potential to connect mathematics and to one's own humanness – *Hunhu Quilting*. Socially, it may open conversations around cultural symbolism and narratives. Mathematically, it allows for spatial thinking and creativity. See Figure 1 p30.

I engaged in the *Hunhu quilting* activity that was inspired by the ATM Tiles and Tiling book activity *'Choose a tile and see how many new tiles you can make from it' (*Edwards,1992, p.13) and Geoff and Barbara's (2008) communal 'Quilting with year 6' activity.

'Prototype quilt', 2023

Thread, fabric, fusible backing.

Embroidered squares by Abimbola Abodunrin, Nihaya Jaber, Qiao Dai, Naihuei Hsieh, Julie McAdam and Mindy Ptolomey.

Figure 1: Quilting for making anticolonial futures (University of Glasgow, 2023)

First, I had to decide what the 'whole' would be; myself. I am bureaucratically labelled as 'Mixed - other'. In attempt to rehumanise myself beyond this label, I used mathematics to calculate my exact ethnic heritage and represented this in 3 colours finding a common denominator of 16, which meant folding paper to form 16 triangles.

Figure 2: Block A, 'Separatism'.

- Yellow: Black African heritage – $\frac{3}{8}$ or $\frac{6}{16}$ (6 yellow triangles)
- Blue: White British heritage – $\frac{1}{2}$ or $\frac{8}{16}$ (8 blue triangles)
- Red: Asian heritage - $\frac{1}{8}$ or $\frac{2}{16}$ (2 red triangles)

Once I had my pieces and base white paper, I found myself grouping the colours together whilst thinking about my family's colonial and anti-colonial history. My mother lived in Southern Rhodesia in a system of separatism. As a category C 'Coloured', at school she learnt, played and ate separately to her white and black peers. Block A represents the separatist system with a separation of each colour.

With my 16 triangles, I decided to follow this narrative and represent the colony of Southern Rhodesia's move to Independence becoming 'Zimbabwe' in 1980. The yellow triangles form an irregular hexagon reminiscent of a shield.

Here I started to experiment with cutting the red and yellow triangles in half forming $\frac{2}{32}$ rather than $\frac{1}{16}$. This block represents a convergence and fragmentation of cultures that my mother experienced when she migrated to England in the late 80s.

Figure 3: Block B, 'Independence'.

Figure 4: Block C, 'Convergence'.

This is where I let my mathematical creativity flourish. If I can cut the $\frac{1}{16}$ pieces in to $\frac{2}{32}$, what if I cut them again in to $\frac{4}{16}$ and then $\frac{8}{128}$? This choice meant that I could create more complex configurations. I started by creating 4 figures (highlighted with a back outline below).

Figure 5: Block D, 'Preservation' view 1.

Figure 6: Block D, 'Preservation' view 2.

There is a tall trapezoid figure made of yellow and red triangles to represent my mother. She is wearing a red *'Dukee'* (headwrap) and carrying my younger sister on her back in a '*Babula*'. My sister is a square, composed of the same triangles as me. I am the square in the front (left) holding on to my mother's

'*Tundela*' (wrapped skirt). The square figure at the back (right) is my older adopted sister, who has a slightly different proportion of coloured triangles here to reflect her own distinct heritage. This block is about preserving aspects of one (or more) cultures whilst also embracing and living within another culture.

Binding it together

What do you notice?

What do you wonder?

Are all the parts from the original quilt block represented in each block?

This activity acted as a 'way in' to rehumanise by connecting mathematical thinking and social storytelling. It is hard to distinguish where the mathematical ends the social begins.

Figure 7: All 4 block.

Final musings

In this article I turned inwards and began by outlining my most recent musings and past reflections about decolonisation and mathematics teaching. I defined decolonisation and the role that mathematics has historically played in the dehumanisation racialised groups. I presented rehumanising mathematics as a 'way in' to decolonising the mathematics curriculum. The reflection of my own attempts to rehumanise mathematics were honest and only highlighted the complex nature of decolonisation and mathematics. By turning outwards to research, I have been inspired to think about multiple 'ways in' to decolonise the mathematics curriculum, quilting being one such way in and a philosophy built on an appreciation of individuality also. I found the '*Hunhu quilting*' activity to be very affirming and cathartic as well as mathematically challenging! I am looking forward to sharing this activity with the children and adults I work with and would encourage you to try the '*Hunhu* Quilting' activity and consider the intricate, recursive connection between mathematics, one's own humanness and the humanness of others.

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Selecting appropriate contexts for undergraduate statistics students

Katie Severn describes how she creates equitable scenarios when teaching statistics.

As a statistics lecturer, I am aware when teaching undergraduate mathematics students, the contexts or scenarios that I use will influence the mathematicians many of them go on to be and could be directly linked to their future careers. Choosing a useful scenario isn't always straightforward as the context needs to be simple enough that it is not distracting from the underlying mathematics but hopefully interesting enough to motivate students into seeing why what they are learning is useful. I also try to consider how the different students in the class might perceive the scenarios and what subtle influences or messages the choice of context might have that is more than just the mathematics being learnt. To ensure we are teaching mathematics for all, and that the legacy of our classes is to form a generation of equitable, ethical and fair mathematicians, the scenarios we choose have to consider more than just the underlying maths.

Two sample hypothesis testing

I am going to use some example scenarios to demonstrate some of the considerations when creating or selecting a 'good' equitable scenario. As a statistics lecturer when creating a scenario, I am usually choosing where the data is from and the questions the data is trying to answer. An instance of this is when teaching two sample hypothesis testing for population mean. In two sample hypothesis testing there are two populations, A and B, and there is a sample of data from both populations measuring a certain thing, C. The aim is to test if the population mean of C is equal for population A and B. To construct a scenario in two sample hypothesis testing you need to choose the populations, A and B and the measurement of interest C. Some scenarios could be:

1. A is company A and B is company B who both create lightbulbs, C is the length of time a lightbulb lasts before it needs to be replaced. The hypothesis test is therefore testing if the mean duration of the lightbulb is the same for both company A and company B.
2. A is the population of lecturers and B is the population of Olympic athletes, C is the time to run 100m. The hypothesis test is therefore testing if the mean time to run 100m is the same between lecturers and Olympic athletes.
3. A is the population of Manchester City players and B is Nottingham Forest players and C is the goals scored by a player in the premier league. The hypothesis test is therefore testing if the mean number of goals scored is different between players in each club.
4. A are the men doing an undergraduate degree in mathematics and B are the women, C is their heights. The hypothesis test is therefore testing if the mean height of women and men undergraduate mathematicians is different.

So, what, if anything, is wrong with these scenarios? Which should I choose to use with undergraduate mathematics students?

Scenario 1: Lightbulb companies

I purposefully chose this scenario to have little influence or personal bias, and partly because of this it is relatively uninspiring. It uses made up companies and an inoffensive product (lightbulb) and so has little meaning. It would be relatively easy to make this example more interesting by considering what students are interested in or what careers they may be going in to and what is currently relevant. Instead of lightbulbs perhaps mobile phones would engage more students? I often think about scenarios related to medical questions (treatment versus control), environmental issues (differences in the climate now versus past) and business analytics (consumer behaviour with versus without marketing initiative), as many students will go into these fields. For the example as it stands, however, I would assume diverse students would interact and relate to this scenario in a similar way, probably without interest, and so it seems equitable at least.

Scenario 2: Lecturers and Olympic athletes

This scenario use populations with more meaning. Students will interact with lecturers constantly, but, as I have chosen such an extreme comparative population it does not seem controversial. In fact, as they are so extreme again it is rather uninspiring. The answer feels like it should be obvious. Olympians, we would expect, are far better athletically then lecturers.

However, because of this obvious fact it is important that if I use simulated data (I make up some times), the data I create needs to fit with our expectations. That is that Olympian times are on average faster. For this scenario I have assumed all students would, if they stopped to consider it, have the same expectation and so again the scenario feels equitable, but this would not always be the case.

Scenario 3: Goal scoring football teams

This scenario concerns me from an equity perspective. To begin with it is more likely to be of interest to a certain demographic. This in of itself isn't a problem, it would be hard to find a scenario that everyone found interesting, it only becomes a problem if all scenarios are aimed at a certain demographic and not to others. As someone self-professed to be 'not into sports', if a large number of scenarios are on sport I will quickly become bored. Similarly, if all scenarios were on animals I would be very happy, but I would imagine a lot of my peers would not be as keen. So, variety is key. However another problem can occur, much like scenario 2 students may have expectation of the answer; the problem this time though is that this will differ between students. If you 'know football' you would expect Manchester City to have a higher mean; this would help you sense check your results. Even those of us in the UK that are not interested in football may well know enough to have a feeling that Manchester City appears to be a more successful club, which probably equates to more goals. However, international students might not have those same expectations and hence ways of sense checking. When thinking of scenarios, it is important to think what additional knowledge a student will bring based on their own experiences and characteristics. Often these will not give them any real benefit when answering a question, but occasionally they will and at that point the scenario is no longer equitable (Boaler, 1993). Of course, how the scenario is being used also matters. If I go through a scenario in a lecture then I can provide a commentary and address the sense checking but if it was an examination question, this would not be possible.

Scenario 4: Men and women studying undergraduate mathematics

This is a scenario I have seen used a lot and reflects the fact that when people are picking binary categories sex and gender are some of the first to come to mind. This means it is a common choice to choose males/men versus females/women when choosing two populations to compare. Of all the scenarios I have listed this one is by far the one I would like to challenge the most, due to multiple issues within this scenario. One issue is often male/men, female/women and sex/gender are all incorrectly used interchangeably when they really represent different things [Thornton et al., 2022]. Sex and gender are not the same and to imply otherwise is not acknowledging our trans students, whose gender is not the same as their sex assigned at birth. Ensuring we use sex and gender and the surrounding terms correctly can go some way to making trans students feel more comfortable and included. I tend to focus on gender and hence use terms men and women, as I believe that is what most scenarios are referring to - as the data usually is self-reported gender and there are very few scenarios where sex is actually relevant. I also make it clear when I use terms men and women I include all those identifying with these, to be inclusive. I also try and point out particularly older textbooks may not be reflective of our understanding of sex and gender now: at the very least this allows students to see they have an ally in me and feel included in my classes. Even if we do get sex and gender correct the choice of C, the measurement being compared, can still lead to lots of issues.

Using gender in statistics scenarios

In our scenario, C being height is relatively unoffensive, being taller or shorter isn't particularly seen as making you more or less superior. But many scenarios are far more controversial with clear implicit biases. When searching on the internet for 'two sample hypothesis test scenario questions' one of the top hits contains a list of questions to practise, and within the list are many questions comparing men and women - including testing if men are more likely to pass their driving test first time, if women do better in English essays and if women take more English courses at college. These all exacerbate harmful stereotypes, like women being perceived as worse drivers or women being better at non-STEM subjects. This is bound to affect the experience and sense of belonging of women learning mathematics, who are already a minority, later in the pipeline (Noyes et al., 2023). It is exacerbated when you consider the fact that the majority of lecturers who will be using these scenarios will be men (Hesa, 2024). Even if the results of the hypothesis test show these hypotheses are not true there is an underlying idea that they could have been, which is why it is worth testing them. You could choose to flip your hypotheses, that is show men are less likely to pass their driving test first time, but this could equally alienate men in a class and pitting genders against each other is rarely helpful for equality. I personally prefer to avoid any scenario that could be seen to promote the battle of the sexes/gender debate unless I am caveating it and

using it to make an example of the limitations of the data analysis.

The times I may choose C, the measurement, to be slightly controversial is if a main aim to enhance students' skills in critiquing data analysis and so I would ensure time for discussion of the limitations and what the findings mean. For the English course scenario, if the data did show women take more English courses, I would facilitate a discussion on what we can conclude. We could conclude that there is significant evidence that women take more English courses then men, we could not conclude women are better at English then men or women prefer English over other subjects. We could not even conclude women prefer English to men, as they may be picking these courses based of stereotypical influences or discrimination in other courses. By using an example that I would usually not be happy with, I have emphasised to the students how the interpretation of their results is very powerful and statistics is more than just numbers and facts. When I do include a scenario with personal characteristics such as gender, I also try to use real data. This means my own biases cannot influence the results. Again, for the English course example, if you were to simulate data on the number of English courses men and women take then if you are a woman, like me, that always favoured STEM subjects you are perhaps likely to simulate data with no difference between men and women, whereas perhaps if you are a man with few women colleagues and you see more women in language departments you may be more likely to create data with a difference. If instead you found real data, this removes the, potentially harmful, influence of your own biases.

Extensive use of scenarios with men and women propagates binary thinking around gender. Some scenarios will explicitly have data where 'not a man' is equivalent to being a woman, which completely removes the possibility of a non-binary person. This devalues any non-binary students in the class and carries powerful but harmful messages for all about gender being binary within society.

What scenarios should we use?

So, what should we do? We could stop using scenarios all together as it requires some thought to get right. But we know scenarios are useful to motivate students and often practical scenarios of the mathematics in action can help understand the underlying theory. Alternatively, we could continue using unfair scenarios and accept they aren't fair - but surely, we should be actively seeking to improve social justice through mathematics education including within our examples. The good news is many scenarios that we already use will be fine, but what we need to do is consciously check they are. When I review existing material to adapt and teach, I will always go through the scenarios and think of students with a wide range of characteristics and think how they could perceive the scenario. This allows me to flag scenarios I want to change as I feel they are problematic. By checking through scenarios, I can also check the variety of contexts covered in the scenarios to avoid an imbalance, for example, mostly sport scenarios. When a new scenario is needed, we can strive to make them equitable like scenario 1, although hopefully a little more motivating. For scenarios that do appear unfair thought needs to be given how they could be taught in a considerate way; or, better still, how they can be leveraged to teach students to critique not just mathematical methods but their application in context.

There is no set of rules to creating the perfect scenario, but I do believe that we need to be aware that our go-to scenarios will be influenced by our own biases and students may relate differently to a scenario then how we intended. Hopefully just by being aware of some of the pitfalls when choosing scenarios will help us avoid them and continue to evaluate and improve the ones we choose.

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Teaching mathematics for social justice: Perspectives of primary PGCE students

Anisha Dhillon, Gwen Ineson, Balbir Kaur, Amaju Ogun and Rosie O'Shea reflect on their experiences of developing mathematics for social justice on an initial teacher education course.

Two of the authors of this article (Gwen and Balbir) are mathematics tutors on a primary PGCE programme and we have been running a module as part of the programme called Teaching mathematics for social justice (TMSJ) for a number of years. We invited some of our alumni, who are now early career teachers, to share some reflections about their experience. The module involved the equivalent of four full days, where we focused on conceptions of social justice, and how it might relate to mathematics teaching. We have been particularly guided by the work of Gutstein (2004) and Wright (2016) that we used to model a possible approach to considering social justice issues in primary mathematics lessons. As part of this module, students completed an assignment that involved planning and teaching a sequence of five mathematics lessons, incorporating principles of teaching mathematics for social justice. This included an opportunity to reflect on the impact of this work on themselves as developing mathematics teachers, their pupils, and the wider school community.

Each of the alumni below were invited to write about their experiences using the following prompts:

- What is your understanding of teaching maths for social justice?
- What maths activities did you use that supported TMSJ?
- How did children respond to your TMSJ sessions?
- What advice would you offer schools about integrating TMSJ?

In the extracts below, our ex-students have written responses to these questions, and they have included examples of work that they conducted while on their final teaching practice.

Anisha:

Teaching mathematics for social justice is a powerful concept that bridges the gap between mathematical learning and the pursuit of a more just society. By addressing global social issues within the context of mathematics education, children not only enhance their knowledge of mathematical concepts but also deepen their understanding of real-world problems. This approach fosters greater engagement and attainment in mathematics by making learning relevant and meaningful as students see the relevance of their education by addressing societal issues.

My research study focused on the topic of money and its connection to the wider global issue of food poverty. I employed role-play activities to help pupils understand financial concepts. Pupils collaborated to calculate costs using addition and subtraction, working with various items in simulated scenarios. For example, pupils might simulate shopping scenarios where they needed to add up the costs of different products or determine the amount of change they would receive. This hands-on, collaborative approach to problem-solving not only clarified mathematical concepts but also brought social issues, such as food poverty, into sharper focus.

Using physical manipulatives in the lessons further enhanced the learning experience. Students interacted with real objects and visual aids (coins and string beads) that deepened their understanding of mathematical principles. For instance, when calculating costs, students could physically handle play money or price tags, making abstract calculations more concrete. Additionally, this approach allowed students to draw on their personal experiences. Those who had visited food banks could relate their own experiences to the mathematical problems being solved. This personal connection fostered empathy and a greater understanding of why some families struggle with poverty.

Students gained insight into the role of community resources in addressing food poverty. By learning about local charities, free school meal programs, and places of worship that support those in need, students saw how their community works to combat social injustice. This experiential learning helped them understand the broader implications of food poverty and encouraged them to think critically about how they could contribute to making a difference.

As the lessons progressed, students began to adopt an activist mindset. They reflected on the importance of school meals and the impact of food waste,

considering how their actions could affect others. This reflection cultivated a sense of responsibility and compassion. Students were not only learning maths but also becoming more socially aware and motivated to act on behalf of others in their community.

My study highlights the significance of integrating teaching mathematics for social justice (TMSJ) into school curriculum. This approach enriches students' understanding of social inequalities while simultaneously enhancing their engagement with mathematics. By connecting mathematical concepts with real-world social issues, TMSJ helps students become more invested in their learning. Schools that adopt this method can expect to see students who are not only more proficient in mathematics but also more empathetic and socially conscious. These students are better prepared to contribute positively to a just and equitable society, demonstrating the profound impact that a socially aware approach to education can have.

The images below provide examples of some of the work that the pupils completed.

Figure 1: A pupil calculating the change from buying a £2.50 bag of pasta.

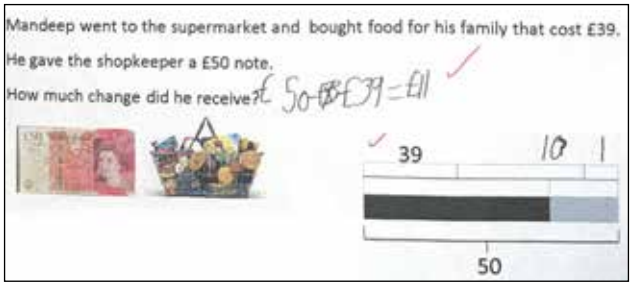

Figure 2: An example of using the bar model to support calculations about shopping.

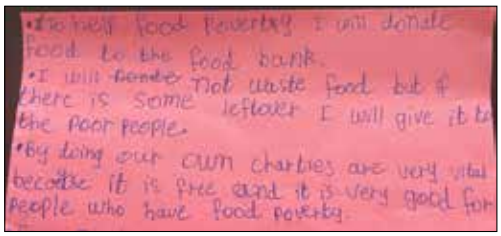

Figure 3: A pupil's reflection about some approaches to addressing food poverty, following an encouragement to take an activist stance.

Rosie:

My understanding of teaching mathematics for social justice is creating a classroom environment where students can connect their educational experience to their society and world around them. TMSJ inspires students to take more of an active role in their education rather than a passive one. TMSJ provides students with the ability to realise that their moral and political actions as an individual and, most importantly, a consumer have real world consequences. Thus, when TMSJ is properly orchestrated in schools, it can act as key stepping stone for introducing students to the world around them in a political sense. TMSJ encourages students to care not only about social issues that affect themselves, but other people as well – including their classmates or people from other walks of life (in different social classes or countries).

The activities I used to support the TMSJ project all revolved around the topic of the accumulation of daily household waste and landfills. My project was titled, *The weight of our rubbish in the classroom*. Together, we looked at the journey our waste takes, mainly food packaging and used containers, from the classroom to the landfill and how it negatively affects our environment. I worked with a class of six- and seven-year-olds and the topic was weight. The project spanned four lessons. My objectives were:

1. To weigh and compare the weight of objects in kilograms (using balance scales)
2. To interpret scales in grams and compare weights of objects measured in grams (using measuring scales)
3. To estimate the weight of an object
4. To apply addition, subtraction, division and multiplication to the context of weight
5. To solve problems through investigation

I felt the children reacted very positively. There was an emphasis on group work throughout the lessons. The children were excited about the topic after the first lesson and it inspired lots of debate around climate change and our roles as individuals in making a difference. After the third lesson, I asked the children to go home and estimate how much rubbish was in their household's rubbish bin. The children were excited to return the next day with their answers and compare with their classmates.

The advice I would give to other educators embarking on TMSJ would be to centre your approach around collaboration and communication in the classroom.

Figure 4. Students using the balance scales.

As a teacher, you should obviously do your best to educate yourself on your chosen topic beforehand. However, you should remember that your students, in turn, will most likely end up teaching you something new about the social justice issue. Another thing I think is very important to remember when integrating TMSJ is to know your students and their backgrounds and cater your lessons to them. Some social justice topics require sensitivity and compassion when approaching the subject.

Ama:

What is your understanding of TMSJ?

TMSJ is a pedagogical approach that connects mathematics with social justice issues. The purpose of TMSJ is to empower students to become critical thinkers and agents of change within their communities. Historically, mathematics has been perceived as a neutral subject detached from societal implications, but TMSJ challenges this traditional view and highlights its relevance to real world issues. Through using this pedagogical approach, educators can teach their students how mathematical tools can be used to interpret social inequities within communities. The aim is that by becoming more conscious, students develop critical consciousness, equipping them with the tools to begin to question social structures, thus allowing them to become empowered to take an activist stance. Gutstein (2003) suggests that developing critical consciousness involves using "Maths to write the world." Furthermore, Gutstein (2003) argues that social justice for mathematics should include three main components: fostering socio-political consciousness, cultivating a sense of agency and promoting positive and cultural identities (2003).

What activities did you use that supported TMSJ?

I taught a group of five- and six-year-olds, and to help them to understand social inequalities, I began by reading a story called *It's a No Money Day* by Kate Milner (See Figure 5), a story about a mother and daughter who need to use food banks. I found that this was an age appropriate approach that effectively allowed the children to understand the purpose of a food bank and the story's visuals provided a clear picture of what a food bank looks like and its purpose. After the children heard the story, they expressed their empathy for the characters in the story.

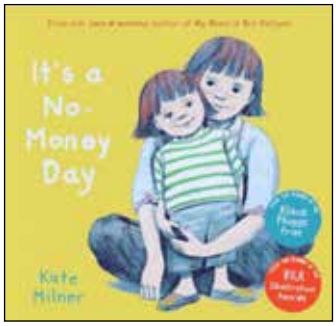

Figure 5. Book used for the work: *It's a no money Day*, by Kate Milner.

In the following lesson, the children participated in a role play where they pretended to be food bank workers and divided food equally between boxes (see Figure 6), as a result of listening to the story, engaging with the role play and learning about food banks in their local area inspired the desire to donate unwanted foods from home to help people like the characters in the story "*It's a no money day*". They also completed a worksheet that asked the children divide quantities of food into equal groups of 4. Figure 6).

Figure 6. Example of work pupils completed.

The book played a crucial role in helping the children understand food banks and empathise with the characters, as they learned about the struggles and realities of relying on one. Reading the story beforehand made the role play activity more impactful and meaningful.

How did children respond to your TMSJ sessions?

The children's responses to the topic exploring food poverty indicated that they enjoyed learning about fractions. The role-play, which allowed them to become workers at a food bank, provided an effective method for understanding fractions. Their feedback at the end of the project showed that they felt a strong sense of empathy particularly towards the social inequalities within their community, the local food bank. The story helped them to grasp the concept of food banks. Although the children at this age were not able to articulate their thoughts in great detail on inequalities, the project planted seeds of awareness that was evidenced by the deep empathy they felt at the end of the project and their desire to donate to food banks.

What advice would you give to schools?

Ensure that the lessons are designed to engage the age group effectively. Role play has proven to be an effective method in supporting younger learners (Maier, 2002), so incorporating this method for your learners will strengthen their understanding. Secondly making the content as relevant as possible to the children is key to holding their attention. This can be achieved through using their names in questions, or learning about their culture in advance, such as incorporating their foods into activities, making it more meaningful and relatable.

Concluding remarks

The main purpose of this article was to share the "real" stories of how beginning teachers raised their awareness and created space for TMSJ in their primary classrooms. Through the process of engaging in research-based practice, these early career teachers have expressed different ways of imagining mathematics education. To deepen our understanding of TMSJ, we, as teacher educators, have also engaged in reciprocal learning alongside our beginning teachers and as a group, we have picked out some key messages to conclude and summarise this article. Firstly, teaching mathematics is not a neutral activity, and mathematics classrooms are part of larger social and political contexts. Ama, in her understanding of TMSJ, expressed this point most effectively when she spoke about using mathematics as a tool to address inequalities and explore social justice issues. Secondly, teachers integrate TMSJ into the real-life experiences of the learners. This approach enhances the meaningfulness of mathematics and promotes the development of mathematical understanding, as illustrated by both Rosie and Anisa, who made links to the home and local community. Finally, teachers need to experience social justice maths lessons as learners themselves to get a sense of how the theory connects with practice.

We wish to end this piece with an invitation to all teachers of mathematics to rethink the way that maths is taught in their classrooms and find forums, spaces, and places to share good practice that advocates for diversity, inclusivity and social justice. Please do share your memoirs.

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What would a socially just mathematics curriculum look like?

Pete Wright shares his vision of a mathematics curriculum that addresses social justice.

Social justice is of primary concern to most student teachers I have encountered in my role as a teacher educator in London and Dundee. Indeed, a commitment to social justice is at the heart of the *Professional Standards for teachers in Scotland* and all teachers in England are expected to consider spiritual, moral, social and cultural aspects of learning. And yet, it is all too easy for experienced teachers to lose sight of their motivation for becoming teachers in the first place due to pressures and constraints they face in the classroom. In this article, I consider practical steps that teachers of mathematics (across all phases) can take towards establishing a socially just curriculum.

An important point to make is that how you teach mathematics is just as important (if not more so) as what you teach. It is too easy to rely on transmission-style approaches to learning in which the teacher models a limited set of procedures and students practise these by applying them to a series of almost identical closed questions before being tested with more of the same. There are several problems associated with this approach. Firstly, the lack of meaning and relevance to students' real lives results in many students becoming alienated and disengaged from the subject. Secondly, students may acquire a series of disconnected pieces of knowledge, but they fail to appreciate their purpose, how they relate to each other or how they can be applied to solve real-life problems.

It has always baffled me why so many teachers fall back on such methods. Some people argue it is because of the pressure to 'teach to the test'. But surely the best way to prepare students for examinations is to help them to develop a deeper understanding of mathematical concepts and an appreciation of how to apply them. The biggest problem students face in an examination is how to identify the mathematics they need to apply in solving an unfamiliar problem. Perhaps the continued acceptance of teacher-centred approaches is down to some students who will always succeed (in exams) regardless of how they are taught, mainly due to the positive dispositions they already have towards learning. Most teachers of mathematics are likely to fall into this group themselves. But, even for this group of students, we are not preparing them to use mathematics productively in their future lives.

A socially just mathematics curriculum is one that motivates all students to achieve whatever their background and prior dispositions towards learning. It would embrace the following elements:

- Collaborative and problem-solving teaching approaches that engage all students and promote mathematical sense-making.
- Using mathematics to explore real-life issues relating to students' experiences that generate greater understanding of their own situations.
- Carrying out mathematical investigations that generate students' collective agency enabling them to take part in future social action.
- Challenging common myths surrounding school mathematics and opening up to scrutiny what it means to be successful.

So, what would this look like in practice? The following example, based on an activity developed as part of a research project carried out with two teacher researchers (Wright, Fejzo, and Carvalho, 2022), will help to explain.

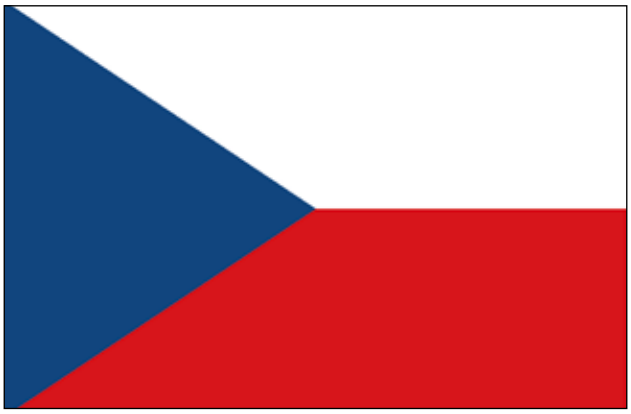

Figure 1: Flag of Chechia.

Students are given the flag of Chechia (Figure 1) printed out on paper and invited to work out the fraction of the flag that is blue, white and red. Pause and consider how you would approach this problem yourself and then how you would normally introduce

it to your students. Then consider these questions:

- Would you need to make some measurements?
- Would you do some folding of the paper? Why?
- Would your subsequent reasoning be different dependent on whether you measured or folded?
- How many different methods can you find to do it?
- Which of these methods is better and why? Are they more efficient? … more accurate? … easier to explain? … more creative?
- Where else could you use these methods?
- What other questions could you ask about this flag?

The best way to take advantage of the richness of this problem is to allow students to work together in small groups and use the above questions as prompts. Encourage students to explain their ideas to you and to each other. The skill is to hold back and allow students to take the lead, using the prompt questions only when they are needed to move the discussions forward. You might need to establish some ground rules to encourage all students to participate and to listen to each other's ideas. Give praise for what you think should be valued most, for example, asking a good question, providing a clear explanation, taking time to explain something to another student, rather than what would normally be valued most in the classroom (producing correct answers in the shortest possible time).

This is where you can start to challenge common myths (see Figure 2) surrounding mathematics, for example, that mathematics is an individual activity or that it is based on right or wrong answers. You could ask questions such as:

- Why is it good to find different ways to solve the same problem?
- How can you benefit from working collaboratively with others?
- How is making errors beneficial?

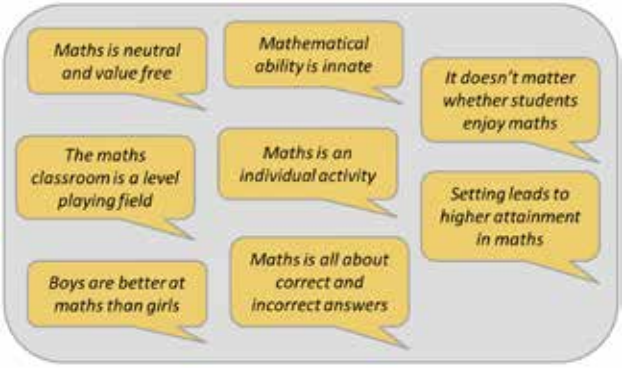

Figure 2: Common myths surrounding mathematics.

Importantly, do not be put off trying out approaches like this with students who you feel will not readily engage. I would argue that one reason why many students have become alienated from mathematics, and hence more likely to exhibit poor attitudes and behaviour, is the prescriptive way that mathematics has been presented to them. I think one of the best strategies available to you to try and re-ignite their interest in the subject is this collaborative approach to learning.

Do remember, however, that students are often resistant to change and new strategies need to be used persistently before the benefits become apparent. It is also the case that some students, particularly those from less privileged backgrounds, are less likely to recognise what they need to do to become successful and more likely to misinterpret the teacher's intentions. This may be exacerbated by teaching approaches that are less structured, such as those advocated above (Wright, Fejzo, and Carvalho, 2022). Be prepared therefore to share your pedagogic rationale with students so that they understand better how to respond appropriately. You can do this by explaining your reasons for adopting particular approaches, or by asking questions such as, *"Why do you think I got you to explain your reasoning to each other?" or "Why do you think I asked you to come up with your own questions?"*

I am not suggesting that there is no place in the mathematics curriculum for demonstrating procedures that students need to know, such as how to add and subtract fractions. What I am saying is that there tends to be too much focus on this aspect of learning mathematics and not enough on applying these procedures in solving meaningful problems and developing mathematical reasoning and agency. Imagine a situation in which students are encouraged to tackle the flag problem above. By dividing the flag up into 8 equal right-angled triangles, two of which

are blue, they might conclude that the fraction that is coloured blue is 2/8. This can lead to an appreciation of why it is important to be able to find equivalent fractions (so that they can recognise 2/8 as the same as 1/4), to subtract fractions (so they can find the fraction shaded red and white together as 1 – 1/4 = 3/4), and to divide a fraction by an integer (so they can divide 3/4 by 2 to get 3/8, the fraction shaded red). Rich problems like this can be used to demonstrate the need for procedures before they are modelled, thus providing these procedures with greater purpose and meaning for students.

Recent moves in England to promote mastery style approaches to teaching mathematics have the potential to prioritise deeper conceptual understanding over procedural knowledge. However, there are many differing interpretations of mastery and they often tend to neglect the importance of student agency. By introducing mathematical concepts in a pre-determined and carefully scripted manner, students can be denied the opportunity to develop mathematical agency. I would argue that if we are to embrace the wider purposes of education, including the desire to make the world a better place, then we need a curriculum that develops skills, knowledge and dispositions amongst learners that, in future, will help us to address challenges facing our society. Such a curriculum needs to enable students to develop powerful knowledge that includes an understanding of how to apply mathematical ideas to solve real-life problems.

In my experience, learners are keen to engage with mathematical inquiries relating to issues of social justice from an early age. The youngest children can have a keener interest in issues of fairness and equity than older children and it is a mistake to assume that such issues should be avoided when teaching them mathematics. Incorporating social justice issue into the mathematics curriculum helps to make the subject more meaningful and relevant to learners (hence increasing engagement), enables students to develop socio-mathematical agency and challenges the myth that mathematics is neutral and value-free (Wright, Kelly & Hilton, 2024). There are some social justice issues for which a full appreciation is dependent upon significant mathematical understanding, and these are a good starting point for classroom activities. I have included several of these in the ATM book that you are introduced to in the centre pages of this issue.

I would like to finish by stressing again that how you teach mathematics is just as important as what you teach. Be ready to promote discussions amongst students (and colleagues), and to ask questions that can challenge learners' pre-conceived notions about mathematics.

The Teaching Maths for Social Justice Network website (www.mathsocialjustice.org) hosts more activities and resources that link social justice and mathematics.

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Mathematics education for solidarity and hope

Hilary Povey and Corinne Angier explore learning and teaching mathematics from a socio-ecological perspective.

Young people worldwide need to believe that they have the power, working together, to re-fashion our relationship with the natural world and with each other. In this article, we argue that developing such a conviction requires a mathematics education for solidarity and hope and we consider what that might look like.

Context

We write as ex teachers, teacher educators, grandparents, researchers, readers and writers who are grappling with trying to live and work for change in an increasingly unequal world, one existentially threatened by the climate emergency, seismic shifts in technology and even the possibility of nuclear conflict. We are also minority world authors, engaging here with a socio-ecological perspective, acknowledging that some of its most important insights have been 'blindingly obvious' to most of the majority world for centuries.

This article is based on earlier writing (Povey and Angier, forthcoming) where we have argued that:

- Capitalism, colonialism and the climate emergency are inextricably linked.
- The capitalist, colonial project 'others' people who are then deemed unworthy or dispensable, derides philosophies other than its own as primitive or fairy stories and understands the planet as a resource to be exploited.
- Survival requires air, oceans, climate systems and species diversity to be held in common. Currently, a few individuals and corporations 'own' a vastly disproportionate share of the world's resources.
- There is an ecological interrelationship between humans and natural world; we are material beings, belong to nature, exist in its midst and are alienated when the reciprocity between self and nature breaks down as it does under capitalism. We need to collaborate across borders and learn from those who have always been 'response-able'.

Mathematics teachers will recognise that education has become saturated with values of competitive individualism and mathematics classrooms can be experienced as hopelessly constrained by a performativity culture and a dominant purpose of attaining qualifications with a high exchange value. Teachers are made responsible for 'closing the gap' created by inequality and non-recognition and students are responsible for their own 'failure' either through lack of inherent ability or through inadequate mental resilience. Education is not seen as a right, a (public) good, a moral enterprise, a vehicle for nurturing our links with the natural world and making us fully human; rather, it is viewed as a commodity – a thing, a product, something with can be cashed in in the human marketplace.

Why mathematics and mathematics education?

Mathematics formats how we see the social world with political decisions invoking the use of mathematical models presented as neutral, value-free, not subject to question, just practical reality providing training in ethics-free thought. Decision-making is increasingly being devolved to AI systems, underpinned by mathematics, systems in which discriminatory perspectives are frequently embedded.

Mathematical tools are used in the cost benefit analysis and statistical models that have privileged intensive destructive land use and the processing of finite resources into throwaway products. Harnessing mathematics to fight back, for example, through climate models, exposes what has always been true: that only some results are acceptable and decisions are always ethical acts.

Mathematics education might enable learners to notice, unpick and critique this formatting but instead it is appropriated as a mechanism for social selection and as a site for training in rule-following obedience.

A mathematics education for solidarity and hope

We are aware that access to school may be prevented by war; by regimes hostile to the education of girls; by migration due to climate change and conflict; by child labour; or by the inhospitable nature of school causing anxiety and refusal. But here we ask: what are the aims, pedagogies and ethical standpoints that lend themselves to the cultivation of solidarity and hope? We understand hope as encompassing

an expectation of alternatives and a realisation that these were always already existent and can be creatively re-imagined. Working collaboratively with mathematical activity enriched by plurality can contribute to un-learning the "othering" of people and philosophies and foster hope and, reciprocally, political solidarity. We need a mathematics education that will allow learners to have their agency and voice recognised and supported; to engage in collaborative activity within the community of their classroom; to develop a sense of powerfulness, solidarity and hope in and through themselves with others and with the planet; and to experience hope-in-the-present (Povey and Angier, 2021). We suggest some threads that might contribute to achieving this.

First is a decolonisation that places mathematical knowledge in *global and historical and cultural contexts*. The global history of mathematics points to human decisions, the evolution and change within mathematics over time, and demonstrates that things have been different in the past or in other places and therefore might be different in the future. Becoming aware that mathematics is uncertain and human-made can be significantly empowering. There are opportunities in the classroom to consider what happens when people choose to flout the rules and there is sometimes a peculiar power in doing this because it is not expected. Within a broader conception of the nature of mathematics there is room for exploration of the geometry of art, games and puzzles from around the world, the history of systems of number and measure and so on, themes connected to cultural activity and closer to the mathematics present in indigenous cultures.

Second is an approach that seeks, welcomes and values the mathematics within *the funds of knowledge of students*; recognising the contribution a young person brings reinforces their legitimate participation. This knowledge may relate to the practices of non-dominant groups, practices that are not generally even labelled mathematics. Munir Fasheh (pp5-8), a Palestinian mathematics teacher and scholar, provided a compelling account (1990) of his realisation that his mother's dressmaking was also mathematical thus drawing attention to the contrast between the constrained school curriculum and the awareness that the features and foundations of mathematics like those of art, poetry, religion and all other human activities are framed within and grow out of our social existence. Mathematics classrooms that foster creativity and collaboration allow more and different ways of participating. They help dent the paradigm of individual attainment and allow learners to experience the joy and power of combining their ideas. This is a much closer match to living well than working alone in silence.

Third is a pedagogy that affords space for *playfulness and curiosity*. Exercising curiosity (Povey, 2020) is a joyful experience and is a powerful and wide-ranging incentive to search for knowledge; such knowledge is imbued with "meaning-fullness". Curiosity starts from the world around us and allows the world to speak to us and not just vice versa, addressing us, calling for a response, eliciting something in us and taking our breath away. To allow this to happen, we need a spacious pedagogy and an expansive timescape, where we move away from industrial time-consciousness to an ecological time where 'things start to regard us and tell us about ourselves in ways we could not have experienced without such whiling' (Jardine, 2008, n.p.). Playfulness includes questioning the authority of authority with everything "up for grabs"; challenging the rules, the conventions of classroom interactions; recognising that learning takes place in an immediate, often emotionally charged, social world; and asserting the agency of players. Playfulness encourages us to engage with others, to create a community, to be flexible, responsive and open. One powerful way to shift from formality to exploratory playfulness is to replace the classroom with outdoor settings.

Respecting funds of knowledge, fostering curiosity, allowing while-ness so that the world can speak to us and responding with agency and openness all provide a space in which students can develop love for the world (Povey and Angier, 2021). Through this we recognise what it is we want to value and protect, helping us to show the next generation that there is something good in the world, something worth preserving. Love for the world can also be generated by the awe, wonder and enchantment that mathematics can supply, such experiences encouraging a de-centring of an individualistic sense of self. This prompts a move away from 'Are you like us?' towards 'What is it like to be you?', a shift from empathy and identity to solidarity, recognizing and valuing other things without having to identify with them as being like us.

Conclusion

A move towards socio-ecological perspectives is not to be seen as leading to certainty, resolution, consensus or, even, coherence. And pessimism is a legitimate response to the global crisis facing

us. Fighting for change is not going to be easy, comfortable or safe and so maybe those of us who have had lives most characterized by ease, comfort and safety are going to find this difficult. However, we seek the cracks where the light gets in. No dominant ideologies are permanent and those who have been excluded or neglected form the basis for resistance and the struggle for something new. To achieve this, solidarity and hope are needed.

There is no reason to suppose that many (most?) teachers do not appreciate wider purposes for mathematics education or the value of pedagogic approaches different from teaching to the test using direct instruction and "teacher proof" material. They can also recognise the pointlessness of training young people to calculate with speed and accuracy, when that can be achieved so easily by computers, whilst stultifying their mathematical creativity and imagination, features that computers lack. This despite the fact that in some settings teachers have very limited choice, with classrooms regularly monitored for compliance and students required to obey without question "first time, every time". In many settings teachers are able to make creative use of the curiously shaped scraps of fabric left after the pattern pieces that fit the requirements have been cut out.

The pinnacle of mathematics education has become an examination hall of silent fearful children desperately hoping they can perform better than enough other people to secure some kind of future. An alternative vision might be a mathematical version of the West Eastern Divan Orchestra (Homepage - West-Eastern Divan Orchestra (west-eastern-divan.org)), who work with joy and rigour across deep historical divisions. A community of peers who have collaborated to investigate a mathematical question of their devising then sharing their explorations. Such an experience of solidarity might provide a space for re-wilding and the hope that wicked problems can be tackled and that we can live in solidarity and partnership with others across the globe and in reciprocity with the planet.

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Mathematics Education for Solidarity and Hope: A Sample of Existing Resources to Support and Stimulate Your Thinking

Exploring number scripts from other cultures both encourages a global perspective and may draw on funds of knowledge of some learners. An example activity is *Which scripts?* (SMILE numbered set 26:1901-1950 (stem.org.uk) , p37). Smile 0691, 1793, 1875, 1881, 1913, 1937 and 2072 (| STEM) also explore number scripts at varying levels of difficulty. For many learners, the puzzles evoke a playful response.

Patterns and designs with religious or cultural significance can provide motivating historical and global contexts in which to explore geometric properties. For example, Islamic design makes extensive use of reflective symmetry as in *Islamic designs* (24733-Drawing pack 1.pdf (stem.org.uk), p21-22). *Hexagons and stars* (WorkOthers_Islam_hex&stars wksht (stem.org.uk)) and *Tiles on grids* (WorkOthers_Islam_tiles on grids wksht (stem.org.uk)) draw on designs based on equilateral triangles and rotational symmetry. Such designs may draw on the funds of knowledge that learners bring to the classroom *Isfahan* (Isfahan (youtube.com)) can be used to stimulate curiosity and the imagination.

Maps and globes can also be used to address effectively all three pedagogies. *Mapping our world with mathematics* (Classroom materials) includes a variety of activities that can be adapted to your local context, that stimulate curiosity and that draw on historical and global perspectives and learners funds of knowledge.

When in doubt, divide it out by Samara Dhankar

Natthapoj Vincent Trakulphadetkrai introduces the winner of the young mathematical story authors competition.

The Young Mathematical Story Authors (YMSA) competition offers a unique opportunity for young mathematics learners globally to creatively embed their mathematical understanding within story contexts. One such example is the winning story from 2023, titled 'When in Doubt, Divide It Out' by Samara Dhankar (a nine-year-old student from India). Samara presents a short story centred on social justice, using the concept of mathematical division as a metaphor for fairness and equality. Set in the peaceful town of Dividior, where resources are shared equitably among diverse families, the story contrasts this harmonious existence with the chaos in a neighbouring village ruled by greed and inequality. When the town is attacked, the protagonist, Mariam (a young Muslim girl), relies on principles of equal distribution to ensure that everyone is cared for and protected. Ultimately, the story illustrates how mathematical knowledge can be used by society to help foster justice and peace that are achievable through fair sharing of resources.

The YMSA competition, organised by the non-profit MathsThroughStories.org initiative, is the world's first international competition encouraging students aged four to 16 years old to create their own mathematical story picture books. Since its inception in 2019, the competition has attracted entries from around 4,000 students across 260+ schools in 23 countries. The 2025 competition will open for entries on Monday, 6th January 2025, and close on Friday, 21st March 2025. The winners of each age category (4-7, 8-11, and 12-16) will receive cash prizes, as will their respective schools, with sponsorship from Oxford University Press, Harper Collins, and Charlesbridge.

For more information, please visit https://www.MathsThroughStories.org/competitions.html or contact Dr. Natthapoj Vincent Trakulphadetkrai (Associate Professor of Mathematics Education at the University of Reading and founder of MathsThroughStories.org) at n.trakulphadetkrai@reading.ac.uk

CHAPTER 1
Mariam's Dividor: A Happy Place

In a pretty, quaint little house in the town of Dividior, lived a girl named Mariam Abdullah who was passionate about Maths and was always eager to learn new concepts. She was especially intrigued by the concept of Division. Her Greek mother, Iris, who was a math teacher, had taught her how to divide so that she could apply it in her life when needed. Her mother always said:

" When in doubt
Do not fret, do not pout.
Instead, Divide it out ! "

Her Lebanese dad, Omar, was a well-loved baker. Every Friday, he brought home 9 handmade, delectable cookies from his bakery and her mother equally divided them and distributed 3 cookies each to Mariam, Omar and herself.

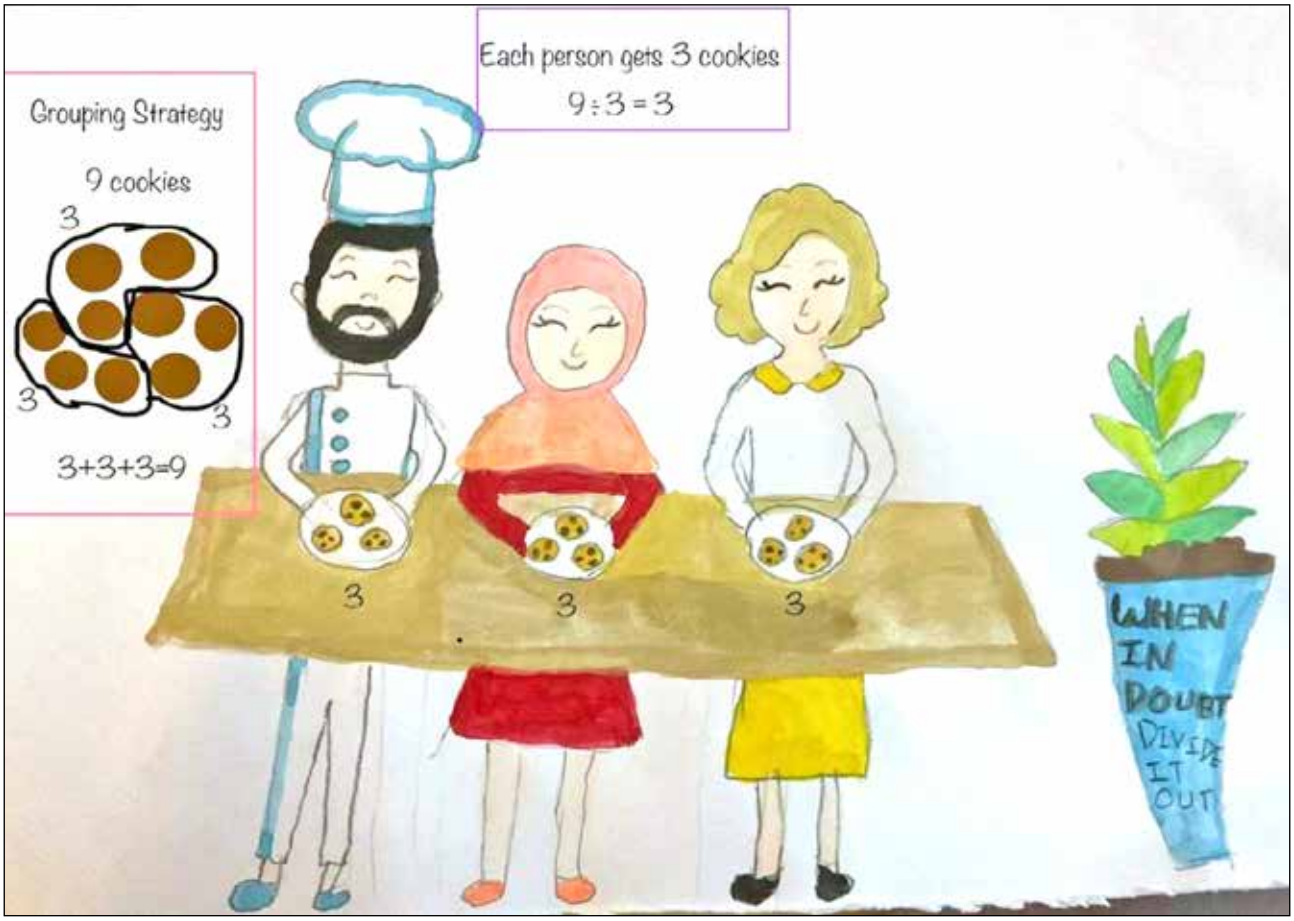


Illustration: Mariam and her family sharing cookies baked by her father.

Total no of cookies: 9

Total no of persons: 3

9 cookies have to be divided equally amongst 3 persons

9 divided by 3 = 3. Each person will get 3 cookies.

Everyone in Dividior was happy because everyone was considered equal. *They believed in equal and fair division of resources. So there was never scarcity. They had equal power, wealth and food. They also believed that everyone was equal no matter what their race, religion or gender was.*

There were 10 families of various origins like African, Japanese, Indian, Lebanese, English, Greek and others living in Dividior. Each family had 3 members only.

Every Monday, a person from each family would go to the Magic Apple Tree which had 60 apples and take 6 apples each for their families. Neither anyone had more nor anyone had less. The Apple tree, this way, never ran out of apples and citizens of Dividior had a balanced and a content life.

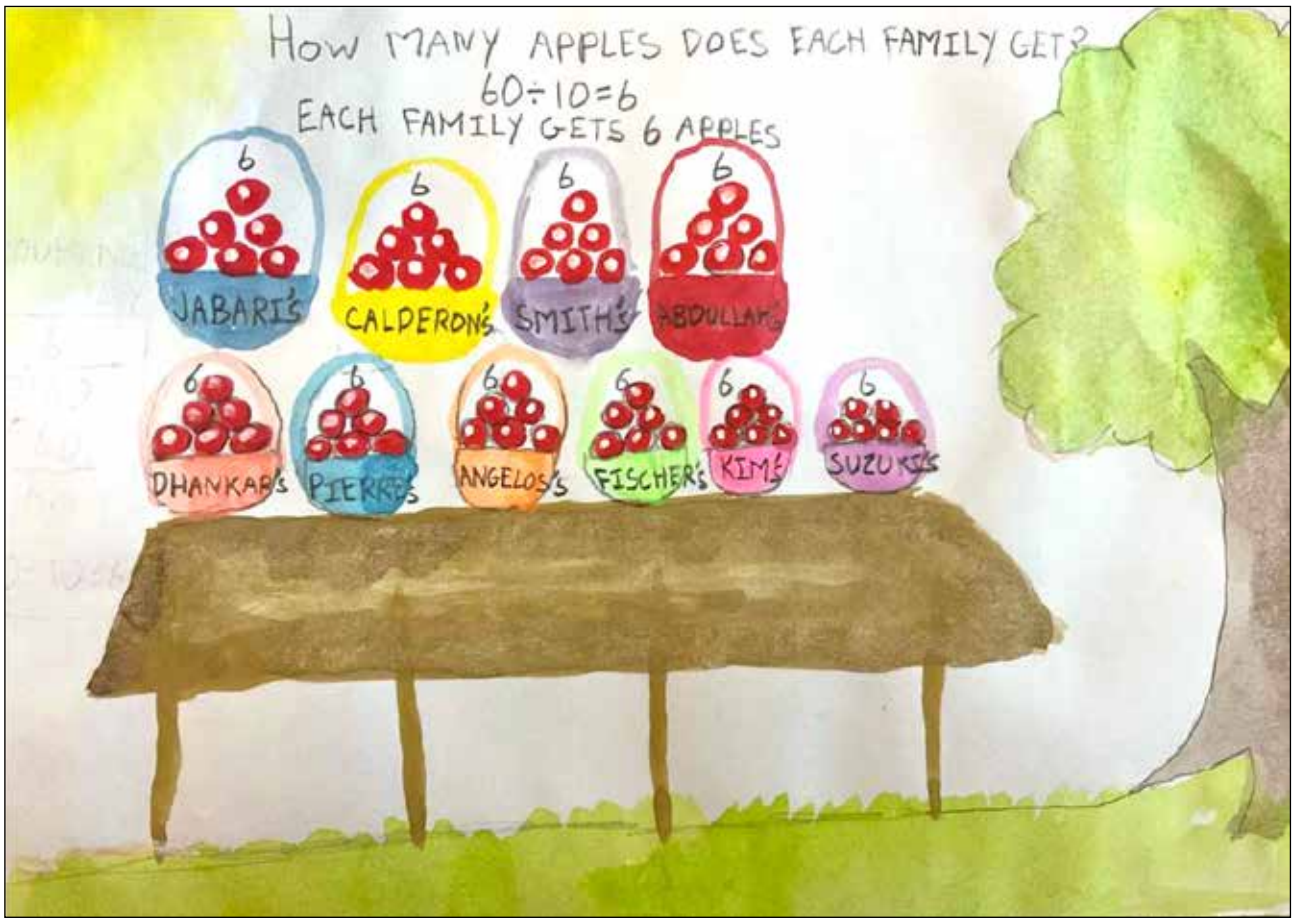


Illustration: The equal division of apples amongst all families.

Total no of apples on the tree: 60

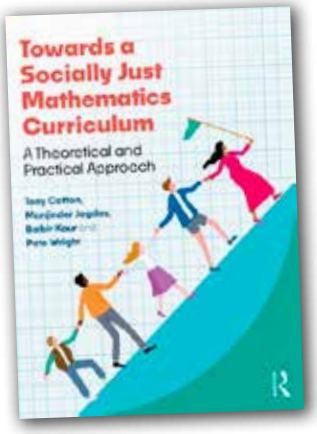
Total no of families: 10

60 apples have to be divided equally amongst 10 families

60 divided by 10 = 6. Each family gets 6 apples.

Book Review

Mahnaz Siddiqui reviews Towards a socially just mathematics curriculum by Tony Cotton, Manjinder Kaur Jagdev, Balbir Kaur and Pete Wright.



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Towards a Socially Just Mathematics Curriculum addresses the urgent need to reshape mathematics education in light of growing global inequities and social tensions. Recent events, such as the summer riots targeting Muslims and asylum seekers, have emphasised the pervasive issues of Islamophobia and racism that plague societies worldwide. These incidents reflect deeper systemic injustices that are mirrored in educational institutions, where marginalised students often face barriers to equitable learning. In the wake of such unrest, there is an increasing call for curricula that not only reflect academic rigour but also promote inclusivity, critical thinking, and social justice, particularly in subjects like mathematics, which can perpetuate or challenge these inequities. I highly recommend this book to schools, Initial Teacher educators, ITE providers, and all practising teachers. It offers a wealth of ideas for embedding purposeful problem-solving and reasoning activities related to social justice into the teaching of mathematics.

I was hooked from the start with a particularly engaging introduction to the authors, which brings a personal touch and invites the reader to connect with their diverse experiences. As a teacher educator based in Liverpool, I have not encountered the same diversity in student teachers as these authors. The personal narratives shared by the authors offer refreshing perspectives and highlight how striving for social justice is often rooted in one's personal philosophy. This rings true to me and a strong determination to address issues of social justice runs through my veins, no doubt inherited from my parents and from lived experiences. As a teacher educator for 21 years, I have consistently prioritised anti-prejudice and anti-racist practices. Before that, as a primary teacher, I endeavoured to incorporate issues of social justice into my teaching. However, over time, I have realised that my practice in mathematics teaching has not always fully aligned with my ideals. This raises an important question: what if that philosophy is absent in student teachers?

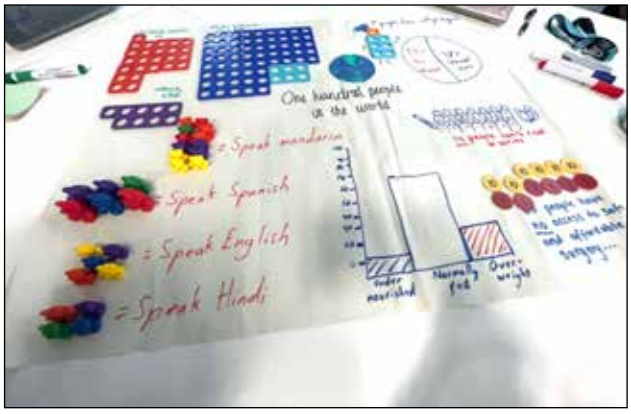
Thompson (2017) observes that student teachers are often reluctant to change their thinking. While encouraging students to become agents of change, initiating conversations about social justice frequently reveals the deep-seated beliefs they bring from their own upbringing. Although changing these perspectives takes time, it is possible, and it must begin early in a teacher education programme, if not before, depending on an individual's life experiences. A teacher education programme should consider how striving for social justice is planned for consciously throughout their curriculum. Thankfully, from recent conversations with undergraduate student teachers regarding embedding social justice issues into 'data handling', I felt assured that they were ready to be the agents of change very much needed for a socially just curriculum. I mention this because student teachers are our future and the strong messages that this book advocates will be in their hands.

I have already taken action by advocating this title to my work collegues and undergraduate teachers as they get ready to embark on their final placement. The book offers practical advice for accessing its content, recommending that those seeking immediate classroom ideas begin with the practical sections before considering the theoretical foundations, but returning to these to make better sense of the underpinning theories and frameworks. My own approach was to read from the start; I will revisit the theoretical chapters to deepen my understanding and integrate these insights more effectively into my teaching. This experience has been an invaluable professional development opportunity, and learning about different frameworks and theory for teaching a socially just mathematics curriculum will empower me to make more informed curriculum choices.

The practical sections bring the theory to life, offering numerous insights for practising teachers and those involved in ITE. I found the age-related

activities particularly useful, although many could be adapted for learners of different ages. The activities are clearly structured, with suggested pedagogical strategies that are easy to follow. The expertise of the authors is clearly evident with tried and tested examples. Readers are directed to enrichment videos and websites for further resources to support their problem-solving investigations and for additional study and research. To fully embrace culturally responsive teaching, further professional development and reading may be necessary. This is acknowledged in the book and teacher participant interviews with Pete Wright highlight the importance of continued professional development and specific work group opportunities such as that offered by Maths Hubs.

One of the activities recommended, which remains a personal favourite, is based on the book, *If the World Were a Village of 100 people*. This exercise can address sensitive topics and reveal misconceptions, such as the percentage of Muslims in the world; these misunderstandings often stem from skewed data or misinformation, particularly from social media and news outlets. This activity highlights inequities in access to education, clean water, electricity and shelter, prompting philosophical discussions that encourage participants to reflect on their own implicit biases and the inequities that exist globally. Incorporating dialogic approaches and philosophical discussions into mathematics teaching can provide students with a deeper understanding of the societal impact of the subject. In the post-COVID era, data has become even more critical in highlighting global inequities. It is my hope that through such educational practices, we can work towards a better world.



Students talked about their chosen representations. As their tutor, I asked prompt questions for clarification or to challenge their ideas. We discussed how discrete data in a bar chart should have a gap. From the data, inequities can be explored and further researched.

In conclusion, this book reminds us that we must continue to challenge the status quo if we want to create lasting change. If we persist in doing things the same way, nothing will improve. In the world that we live in, it is hoped that there is a sense of urgency for our teachers to receive the relevant professional development training to support them on enrichment journeys related to becoming the agents of change needed to combat institutional racism and inequities. If new teachers can take a stand, there remains hope for meaningful transformation. We must have faith in our future student teacher to keep the flame of passion for a socially just mathematics curriculum burning.

Mahnaz Siddiqui (she/her, multi-heritage) is a Senior Lecturer in Initial Teacher Education at Liverpool John Moores University (LJMU), since 2017. Mahnaz is learning more about how to actively be anti-prejudiced.

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Final words

by Tony Cotton

I have found writing the opening and closing section to this issue incredibly difficult. Partly because, I realise, they are literally the final words I will have to write (if not say) on mathematics education. I have promised myself that these are the final words. It is time for the rest of you to take over! However, I also find it impossible to articulate my thoughts whilst my reflective self is invaded by images of war, by the realisation that mathematics is being used to create new ways that human beings can kill each other. By the inhumanity which seems ever present. And I believe this is a failure of education. This is articulated much more clearly by Munir Fasheh in his opening article. But I must have hope. On a visit to an exhibition of war photography by James Nachtwey I read this statement by James about his work.

> If people are afraid, it does not mean they lack courage. If people are suffering it does not mean they lack dignity. Overcoming fear is the definition of courage. The struggle to live, in the face of tragedy and suffering, is an expression of hope. I've seen people with absolutely nothing left but their will to live. They have not given up hope. Why should anyone else give up hope for them?

Similarly, this morning, whilst I waited for a flight out to Belize where I am working with their wonderful teachers, a country which is being hugely impacted by climate change (yes – there is an irony in my flying there!) I read this in my New Statesman:

> True creativity refuses to be altered or extinguished by the violence that is always present at the centre of things. (Rachel Cusk interviewed by Anna Leiszkiewicz in New Statesman 7-14 November 2024)

So, I must hope. I owe that to those who suffer in war. And I must express my hope through my writing. What do I hope my time as editor of Mathematics Teaching has achieved? I hope that, as teachers have engaged with the writing, they have found solace and succour that have given them the energy to be creative. This is what reading Mathematics Teaching gave me as a young teacher. I hope, that in the same way the ATM modelled for me how to teach with the guiding principles at the core of what I do the last nine years have offered similar models for you. Perhaps 'unleashing the power of the learner' makes more sense now.

I am proud that there is a diversity of voices across the nine years. And, I am proud that, on the whole, these are from the fringes of mathematics education rather than the centre. It is at the fringes that exciting, creative approaches to learning mathematics take place. We owe it to our learners to explore education at the fringes of what is possible. I am proud that writers from around the world, from in prison and from the artists' studio have contributed. I have learned much about what is possible and what has been stifling for individuals from the My Maths interviews. And the use of articles from the archive has kept alive the wisdom of the ages and ageless.

So, I do move forward with hope. I must, for the sake of my grandchildren and for everyone's grandchildren. And I leave you as I move away from mathematics education to explore how I can create 'homes of learning'. And, just maybe, as Jalaluddin Rumi wrote, "Beyond right and wrong there is a field; I will meet you there." Where you can tell me about your 'home of learning'.

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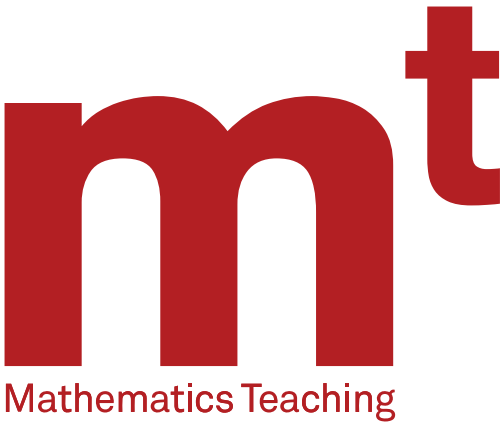
Finally, thanks to my family and friends who surround me with love and fill me with hope.

And, looking forwards, good luck to Sue Pope who takes over as editor from the next issue. Remember Sue, we are only looking after MT for the mathematics teachers who come next.



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Aims of ATM

The Association of Teachers of Mathematics aims to support the teaching and learning of mathematics by:

- encouraging increased understanding and enjoyment of mathematics.
- encouraging increased understanding of how people learn mathematics.
- encouraging the sharing and evaluation of teaching and learning strategies and practices.
- promoting the exploration of new ideas and possibilities.
- initiating and contributing to discussion of and developments in mathematics education at all levels.

Guiding principles

The ability to operate mathematically is an aspect of human functioning that is as universal as language itself. Attention needs constantly to be drawn to this fact. Any possibility of intimidating with mathematical expertise is to be avoided.

The power to learn rests with the learner. Teaching has a subordinate role. The teacher has a duty to seek out ways to engage the power of the learner.

It is important to examine critically approaches to teaching and to explore new possibilities, whether deriving from research, from technological developments or from the imaginative and insightful ideas of others.

Teaching and learning are cooperative activities.

Encouraging a questioning approach and giving due attention to the ideas of others are attitudes to be encouraged. Influence is best sought by building networks of contacts in professional circles.

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